Final report for a MISGAM short visit grant

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Numerical study of the small dispersion limit for highly dispersive equations

The Cauchy problem for purely dispersive regularizations of the Hopf equation with small dispersion parameter $\epsilon \ll 1$ as the celebrated Korteweg-de Vries (KdV) equation,

$$u_t + uu_x + \epsilon^2 u_{xxx} = 0,$$

is characterized by the appearance of a zone of rapid modulated oscillations. For KdV, an asymptotic description of these oscillations was given in the works of Lax and Levermore [LL], Venakides [V2] and Deift, Venakides and Zhou [DVZ], which was numerically implemented in a previous paper [GK]. Dubrovin [D] studied dispersive regularizations of higher order to the Hopf equation which are not exactly integrable, and which contain high order spatial derivatives. Such equations are studied in hydrodynamics and are therefore of practical relevance.
In the numerical study of such equations, the main problem is in the resolution of the strong gradients in the oscillatory region. To do this with the 5th derivatives appearing in the equations the standard double precision computations (16 digits) are not sufficient. During this visit we have therefore started to translate existing Matlab codes to C++. It is planned to use multiprecision calculus in this environment to provide the necessary numerical resolution.

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[D] B. Dubrovin, *On Hamiltonian Perturbations of Hyperbolic Systems of Conservation Laws, II: Universality of Critical Behaviour*, Comm. Math. Phys., **267** (2006), 117.
[GK] T. Grava and C. Klein, *Numerical solution of the small dispersion limit of Korteweg de Vries and Whitham equations*, Comm. Pure Appl. Math., **60**(11), 1623-1664 (2007).
[LL] P. D. Lax and C. D. Levermore, *The small dispersion limit of the Korteweg de Vries equation, I,II,III*, Comm. Pure Appl. Math. **36** (1983), 253-290, 571-593, 809-830.
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