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During the period from 23-03-2009 to 2-04-2009, hosted by Prof. Eugene Kanzieper, I stayed in Yad Hashmona to study the relationship between replica methods and integrability and some more aspects of the link between random matrices, statistical models and integrability. Moreover, in the same period, I had the chance to follow the workshop on *Random matrices and Integrability* as well as the lessons given by Prof. Craig Tracy and Prof. Alexander Its in the framework of Batsheva Fellowship. Replica methods have been used by Brezin and Hikami to establish a very explicit connection between the Gaussian Unitary Ensemble with an external potential and the generalized Kontsevich Matrix Model. It is interesting to observe that, on the other hand, the GUE with external potential is also connected with brownian bridges of N particles starting from a single point and arriving to several points $a_1, \ldots, a_p$. I started to study the possibility to establish, following this path, a direct link between brownian motions and generalized Kontsevich Matrix Model. This is just the beginning of the project and no concrete results have been obtained yet. Still during my stay I learned how the work on replica methods of Prof. Eugene Kanzieper and Vladimir Osipov is related to the Adler-Shiota-van Moerbeke approach on random matrices and Virasoro algebras (see for instance the article *Are bosonic replicas faulty?*) and this link will play for sure a key role in my future work. Some more concrete results have been reached discussing with Prof. Alexander Its a project we already started one year ago. The idea is to use Riemann-Hilbert approach to evaluate the asymptotics of some block Toeplitz determinants arising in the theory of dimer models. Our approach is based on the use of theta functions and it is conceptually related to the well known work on finite-gap integration for solitonic equations. The concrete result reached till know is the RH factorization of the symbol of the block Toeplitz determinant studied by Estelle Basor and Torsten Ehrhardt in the article *Asymptotics of block Toeplitz determinants and the classical dimer model*. It remains to use this RH factorization to recover the asymptotics of the Toeplitz determinant for large N using a formula discovered in the seventies by H. Widom and applied in the recent work of Alexander Its with Kanzieper, Mezzadri and Mo. This work should result in a joint paper with Prof Alexander Its.