

# MISGAM application for a short visit grant

Name: C. Klein

Host Institution: SISSA, Trieste Italy

Hosted by B. Dubrovin

## Description of the proposed project work: numerical study of the small dispersion limit for highly dispersive equations.

It is well known that the Hopf equation

$$u_t + uu_x = 0$$

has shock solutions for generic smooth initial data. The Korteweg-de Vries (KdV) equation can be seen as a purely dispersive regularisation of the Hopf equation whose solutions will show rapid modulated oscillations in the shock region of the corresponding Hopf solutions in the small dispersion limit  $\epsilon \rightarrow 0$ . For the KdV case, an asymptotic formula was obtained in the works of Lax and Levermore [LL], Venakides [V2] and Deift, Venakides and Zhou [DVZ], which was numerically implemented in a previous paper [GK]. Dubrovin studied dispersive regularisations of higher order to the Hopf equation which are not exactly integrable. Such equations are studied in hydrodynamics and are therefore of practical relevance. It is the purpose of this visit to obtain a numerical implementation of these equations, which is very difficult due to the high order of the derivatives. Consequently multiprecision calculus has to be used.

[DVZ] P. Deift, S. Venakides, and X. Zhou, *New result in small dispersion KdV by an extension of the steepest descent method for Riemann-Hilbert problems*, IMRN **6**, (1997), 285-299.

[D] B. Dubrovin, *On Hamiltonian Perturbations of Hyperbolic Systems of Conservation Laws, II: Universality of Critical Behaviour*, Comm. Math. Phys., **267** (2006), 117.

[GK] T. Grava and C. Klein, *Numerical solution of the small dispersion limit of Korteweg de Vries and Whitham equations*, Comm. Pure Appl. Math., **60**(11), 1623-1664 (2007).

[LL] P. D. Lax and C. D. Levermore, *The small dispersion limit of the Korteweg de Vries equation, I,II,III*, Comm. Pure Appl. Math. **36** (1983), 253-290, 571-593, 809-830.

[V] S. Venakides, *The Korteweg de Vries equations with small dispersion: higher order Lax-Levermore theory*, Comm. Pure Appl. Math. **43** (1990), 335-361.

# Christian Klein

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## Research

2007 - present **Professor for Applied Mathematics, IMB, University of Burgundy**  
2004 - 2007 **Researcher, MPI for Mathematics in the Sciences**  
(Prof. E. Zeidler).

2003 - 2004 **Post Doctoral Fellow, MPI für Physik**  
(Dr. D. Maison).

## Education

2002 - 2003 **Post Doctoral Fellow, LUTh, Observatoire de Paris, Habilitation**  
(Dr. E.ourgoulhon).

Habilitation: Paris VI, 4.12.2003

2001 - 2002 **Post Doctoral Fellow, Max-Planck-Institut für Physik, München**  
(Dr. D. Maison).

2000 - 2001 **Post Doctoral Fellow, University of Paris VI**  
Laboratoire de Gravitation et Cosmologie Relativistes (Prof. R. Kerner).

1997 - 1999 **Post Doctoral Fellow, University of Tübingen (Germany)**  
Division of Theoretical Physics (Prof. H. Pfister).

1994 - 1996 **Post Doctoral Fellow, Max-Planck-Society, Jena (Germany)**  
Research Unit 'Theory of Gravitation' at the FSU Jena (Prof. G. Neugebauer).

1991 - 1993    **Ph.D., University of Tübingen (Germany),**  
Division of Theoretical Physics.  
Thesis: *Rotational Perturbations and Frame Dragging Effects*  
*in a Friedmann Universe.*  
Thesis Advisor: Professor Herbert Pfister.

PHD: University of Tübingen, 23.12.1993

1988 - 1990    **M.S., (magna cum laude) University of Karlsruhe (Germany)**  
Department of Physics.  
Master's Thesis: *Decay of a Metastable State under the Influence of Weak Damping.*  
Master's Thesis Advisor: Professor Albert Schmid.

Diploma: University of Karlsruhe, 29.08.1990

1985 - 1987    **B.S., (magna cum laude) University of Karlsruhe**  
Department of Physics.

## Publication List

1. T. Grava and C. Klein, 'Numerical study of a multiscale expansion of the Korteweg de Vries equation', arXiv: [math-ph/0702038](https://arxiv.org/abs/math-ph/0702038), Proc. Royal. Soc. A **464** 733-755 (2008).
2. B. Dubrovin, T. Grava and C. Klein, 'On universality of critical behaviour in the focusing nonlinear Schrödinger equation, elliptic umbilic catastrophe and the *tritronquée* solution to the Painlevé-I equation', accepted for publication in J. Nonl. Sci., arXiv: [0704.0501](https://arxiv.org/abs/0704.0501) (2007).
3. T. Grava and C. Klein, 'Numerical study of a multiscale expansion of KdV and Camassa-Holm equation', arXiv: [math-ph/0702038](https://arxiv.org/abs/math-ph/0702038), accepted for publication in CONM (2006).
4. C. Klein, 'Fourth order time-stepping for low dispersion Korteweg-de Vries and nonlinear Schrödinger equation', accepted for publication in ETNA (2006).  
<http://www.mis.mpg.de/preprints/index.html>
5. C. Klein and O. Richter, 'Ernst Equation and Riemann Surfaces', Lecture Notes in Physics **685** (Springer) (2005).

## Curriculum Vitae of Christian Klein

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Christian Klein got his PHD in physics in 1993 at the university of Tübingen. Consequently he held postdoctoral positions at the universities of Jena, Tübingen and Paris VI, at the observatoire de Paris, at the Werner Heisenberg institute in Munich, and at the Max Planck institute for Mathematics in the Sciences in Leipzig. Since 2007 he is professor at the IMB at the university of Burgony in Dijon.

He has done research on general relativity, integrable systems, Riemann surfaces and numerics with spectral method. He is (co)-author of more than 30 papers. He was a member of the advisory board of the topical group general relativity of the German Physical Society (DPG), and has organized several workshops and schools.

### Selected Recent Publications:

B. Dubrovin, T. Grava and C. Klein, ‘On universality of critical behaviour in the focusing nonlinear Schrödinger equation, elliptic umbilic catastrophe and the *tritronquée* solution to the Painlevé-I equation’, J. Nonl. Sci. 10.1007/s00332-008-9025-y (2008).

C. Klein, ‘Fourth order time-stepping for low dispersion Korteweg-de Vries and nonlinear Schrödinger equation’, ETNA Vol. 29 116-135 (2008).

C. Klein, C. Sparber and P. Markowich , ‘Numerical study of oscillatory regimes in the Kadomtsev-Petviashvili equation’, J. Nonl. Sci. Vol. 17(5) 429-470, arXiv: [math-ph/0601025](https://arxiv.org/abs/math-ph/0601025) (2007).

C. Klein, A. Kokotov and D. Korotkin, ‘Extremal properties of the determinant of the Laplacian in the Bergman metric on the moduli space of genus two Riemann surfaces’, Math. Zeitschr. 10.1007/s00209-008-0314-9 (2008).

T. Grava and C. Klein, ‘Numerical solution of the small dispersion limit of Korteweg de Vries and Whitham equations’, Comm. Pure Appl. Math., Vol. 60(11), 1623-1664 (2007).