

Description of the proposed research project

DISPERSIONLESS 2D TODA HIERARCHIES, INFINITE-DIMENSIONAL FROBENIUS MANIFOLDS AND REDUCTIONS

The dispersionless integrable hierarchies have recently attracted much interest. On one hand they form the basis for the classification of dispersive integrable equations. For example in the Dubrovin-Zhang theory for the classification of bi-hamiltonian hierarchies one considers a topological expansion which has as leading term a dispersionless hierarchy and this is in one-to-one correspondence with a Frobenius manifold. This approach is rooted in the fascinating developments connecting integrable hierarchies with topological field theories and Gromov-Witten invariants.

On the other hand dispersionless equations have proved to be of interest in the theory of conformal maps. Recent developments by Wiegmann, Zabrodin et al. have shown for example that the dispersionless 2D Toda hierarchy describes the Laplacian growth i.e. the dynamics of conformal mappings in the complex plane. Moreover the finite-dimensional reductions of Benney (i.e. dispersionless KP) are strictly linked to deformations of conformal maps to slit domains and to their associated Loewner equations, as in the work of Gibbons and Tsarev.

In these examples it is apparent the central role played by infinite-dimensional hierarchies (e.g. 2D Toda and dKP). Recently, the discovery of an associated infinite-dimensional structure of Frobenius manifold (by Dubrovin, Mertens and myself) gives further interest to these structures. On one hand one is led to consider the behaviour of the reduction with respect to the structure of Frobenius manifold. On the other hand one expects the possibility of finding Frobenius manifold structures associated to many other infinite-dimensional dispersionless hierarchies.

Prof. M. Manas and Prof. L. Martinez Alonso have recently obtained interesting developments in the theory of infinite-dimensional and multicomponent 2D Toda theories. First they have completed the definitions of Ueno and Takasaki by introducing sets of new discrete

flows which were previously observed in the theory of multicomponent KP. Moreover they have considered the non-trivial problem of finding the dispersionless limit of such hierarchies. Not only one can obtain the genus-zero universal Whitham hierarchy in such limit, but one can also find a new type of dispersionless hierarchy which they call multicomponent 2D Toda hierarchy. This raises many interesting questions on the existence of associated Frobenius manifold structures and on the nature of the reductions of such systems and their relationship with the theory of conformal maps.

I am confident that the possibility of visiting Prof. Manas (and Prof. Martinez Alonso) in Madrid will give us the opportunity of developing different lines of research based on the themes sketched above.