## Description of the proposed research project and aim of the visit

### Topological expansions in matrix models and integrable systems

Topological expansions appeared in the theory of two-dimensional quantum gravity as a way to take into account contributions coming from different genera (topologies). The realization of the partition functions for such theories as double scaling limits of certain matrix models gave the possibility of identifying the genus of the surface with the genus of the ribbon Feynman diagram corresponding to perturbative expansion of the matrix integral.

On this basis further developments have shown the remarkable connections with the theory of integrable systems and of Gromov-Witten invariants of symplectic manifolds.

Prof. Eynard has not only obtained explicit computations of the topological expansion of the partition function of certain matrix models, but has recently shown, in collaboration with N. Orantin, how such procedure can be used generalized to obtain symplectic invariants from any regular spectral curve. These partition functions are expected to be generating functions of Gromov-Witten invariants of some toric manifold.

On the other hand, I have been working recently on a certain integrable hierarchy ($N = 2$, $M = 1$ bigraded Toda hierarchy) and in particular on the proof of invariance of the tau-function under the action of (half of) the Virasoro algebra. This work is strictly connected to the Dubrovin-Zhang scheme of classification of bi-Hamiltonian integrable hierarchies. In particular a central role is played by the topological expansion of the the equations, which are regarded as higher-genus perturbations of their *genus-zero* leading term, which is essentially described by a Frobenius manifold.

In particular the Frobenius manifold associated to the $N, M$ bigraded Toda hierarchy coincides with the quantum cohomology of certain $P^1$ orbifolds with orbifold points of order $N - 1, M - 1$ respectively. This leads to the conjecture that a certain tau-function of these hierarchies, the one associated to the *topological solution*, is the generating function for the Gromov-Witten invariants of such orbifolds. This tau-function being uniquely identified by the so-called Virasoro constraints, the Virasoro invariance of the hierarchy is the essential step in proving the conjecture.

This program has been carried out by Dubrovin and Zhang in the case $N = M = 1$ which corresponds to the Extended Toda hierarchy. In this case the target manifold is given simply by the Riemann sphere.

It is clear that recent results of Eynard pose many interesting questions to be addressed from the integrable systems point of view. The symplectic invariants of the spectral curves should have an interpretation in terms of tau-functions for certain integrable hierarchies. On the other hand the bigraded Toda hierarchies define topological tau-functions which could be interpreted as spectral invariants of some spectral curves in Eynard's framework.

I hope that the opportunity of this visit will give my the possibility of understanding better the results of Eynard and of finding possible applications to the theory of integrable systems.