Final report for a MISGAM short visit grant

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Numerical study of the small dispersion limit of the Camassa-Holm equation and the Whitham equations

The Cauchy problem for the Camassa-Holm (CH) equation,

$$u_t + 2ku_x - u_{xxt} + 3uu_x = \epsilon^2(2u_xu_{xx} + uu_{xxx}),$$

in the limit of low dispersion $\epsilon \ll 1$ is as the corresponding limit for the celebrated the Korteweg-de Vries (KdV) equation characterized by the appearence of a zone of rapid modulated oscillations. For KdV, an asymptotic description of these oscillations was given in the works of Lax and Levermore [LL], Venakides [V2] and Deift, Venakides and Zhou [DVZ], which was numerically implemented in a previous paper [GK].
In the CH case, the non-local character of the equation has interesting consequences as the appearence of non-smooth solitons, so-called peakons. In [AG] Abenda and Grava presented the modulation theory for the CH equation, the so-called Whitham equations which are in contrast to the KdV case not strictly hyperbolic. During this visit we obtained a numerical solution for the Whitham equations for smooth humplike initial data to CH. We also implemented the elliptic solution to CH and provided numerical evidence for a conjecture on the phase for the asymptotic solution in the oscillatory region. The scaling behavior of the difference between the CH and the asymptotic solution in dependence of ϵ is being done. A publication of these results is in preparation.

[AG] S. Abenda and T. Grava, *Modulation of Camassa-Holm Equation and Reciprocal Transformations*, Ann. Inst. Fourier, **55** (6), 1803 (2005).
[DVZ] P. Deift, S. Venakides, and X. Zhou, *New result in small dispersion KdV by an extension of the steepest descent method for Riemann-Hilbert problems*, IMRN **6**, (1997), 285-299.
[D] B. Dubrovin, *On Hamiltonian Perturbations of Hyperbolic Systems of Conservation Laws, II: Universality of Critical Behaviour*, Comm. Math. Phys., **267** (2006), 117.
[LL] P. D. Lax and C. D. Levermore, *The small dispersion limit of the Korteweg de Vries equation, I,II,III*, Comm. Pure Appl. Math. **36** (1983), 253-290, 571-593, 809-830.
[V] S. Venakides, *The Korteweg de Vries equations with small dispersion: higher order Lax-Levermore theory*, Comm. Pure Appl. Math. **43** (1990), 335-361.

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Research

2007 - present **Professor for Applied Mathematics, IMB, University of Burgony**
2004 - 2007 **Researcher, MPI for Mathematics in the Sciences**
(Prof. E. Zeidler).

2003 - 2004 **Post Doctoral Fellow, MPI für Physik**
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Education

2002 - 2003 **Post Doctoral Fellow, LUTh, Observatoire de Paris, Habilitation**
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Habilitation: Paris VI, 4.12.2003

2001 - 2002 **Post Doctoral Fellow, Max-Planck-Institut für Physik, München**
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2000 - 2001 **Post Doctoral Fellow, University of Paris VI**
Laboratoire de Gravitation et Cosmologie Relativistes (Prof. R. Kerner).

1997 - 1999 **Post Doctoral Fellow, University of Tübingen (Germany)**
Division of Theoretical Physics (Prof. H. Pfister).

1994 - 1996 **Post Doctoral Fellow, Max-Planck-Society, Jena (Germany)**
Research Unit 'Theory of Gravitation' at the FSU Jena (Prof. G. Neugebauer).

1991 - 1993 **Ph.D., University of Tübingen (Germany),**
Division of Theoretical Physics.
Thesis: *Rotational Perturbations and Frame Dragging Effects in a Friedmann Universe.*
Thesis Advisor: Professor Herbert Pfister.
PHD: University of Tübingen, 23.12.1993

1988 - 1990 **M.S., (magna cum laude) University of Karlsruhe (Germany)**
Department of Physics.
Master's Thesis: *Decay of a Metastable State under the Influence of Weak Damping.*
Master's Thesis Advisor: Professor Albert Schmid.

Diploma: University of Karlsruhe, 29.08.1990

1985 - 1987 **B.S., (magna cum laude) University of Karlsruhe**
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Publication List

1. T. Grava and C. Klein, 'Numerical study of a multiscale expansion of the Korteweg de Vries equation', arXiv: `math-ph/0702038`, Proc. Royal. Soc. A **464** 733-755 (2008).

2. B. Dubrovin, T. Grava and C. Klein, 'On universality of critical behaviour in the focusing nonlinear Schrödinger equation, elliptic umbilic catastrophe and the *tritronquée* solution to the Painlevé-I equation', accepted for publication in J. Nonl. Sci., arXiv: `0704.0501` (2007).

3. T. Grava and C. Klein, 'Numerical study of a multiscale expansion of KdV and Camassa-Holm equation', arXiv: `math-ph/0702038`, accepted for publication in CONM (2006).

4. C. Klein, 'Fourth order time-stepping for low dispersion Korteweg-de Vries and nonlinear Schrödinger equation', accepted for publication in ETNA (2006).
http://www.mis.mpg.de/preprints/index.html

5. C. Klein and O. Richter, 'Ernst Equation and Riemann Surfaces', Lecture Notes in Physics **685** (Springer) (2005).