SHORT VISIT GRANT PROPOSAL

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Poisson groupoid actions and integrable systems

Poisson-Nijenhuis manifolds were introduced by Magri and Morosi in [**3**] and constitute a geometric facet of Gel'fand and Dorfman's algebraic approach to integrable systems [**1**]. There exist a huge number of bi-Hamiltonian systems arising from Poisson-Nijenhuis manifolds. The characterization of Poisson-Nijenhuis manifolds in terms of Lie bialgebroids given by Kosmann-Schwarzbach in [**2**] allows to borrow Lie theoretic techniques to study the complete integrability of the associated dynamical systems; in particular, the study modular classes of Poisson manifolds and Lie algebroids has recently led to a better understanding of the geometry of some integrable hierarchies, thanks to the work of various authors.

In my recent work I used Kosmann-Schwarzbach's description to obtain a reduction result for Poisson-Nijenhuis manifolds under compatible actions as general as those of Poisson groupoids. Other reduction procedures have been studied earlier by Vaisman [**4**] and Magri et alia and represent a powerful method to the study of complete integrability.

The aim of my visit to SISSA is to discuss with Prof. Dubrovin, Prof. Tamara Grava and the members of their research group in order to investigate the chance of applying the reduction under Poisson groupoid actions to infinite dimensional integrable systems; Prof. Pedroni is an expert on integrable systems associated with Poisson-Nijenhuis manifold, the aim of my visit to Dalmine is to discuss special examples arising in that contest.
I recently got interested to symplectic field theory; I also hope to learn more on this fascinating subject during my visit at SISSA.

Visit Report. During my visit to S.I.S.S.A:

- The chance of applying groupoid techniques to the reduction of Hamiltonian and bi-Hamiltonian PDE's has been discussed with Prof. Dubrovin, Prof. Grava and Guido Carlet. After discussing a few examples in the context of the KDV hierarchy, it has emerged that my reduction results could have applications to Hamiltonian PDE with bilocal or nonlocal functional, where a recursion operator can be introduced. We plan to investigate further this issue.
- I gave a talk by the title "Integration of quotient Poisson manifolds" at the Mathematical Physics Colloquium.

During my visit to Dalmine:

- An approach to compute Poisson cohomology via invariant Nijenhuis cohomology has been discussed with Prof. Pedroni and is now matter of joint research with him and his collaborators. A symplectic–Nijenhuis manifold (M, ω, N) gives rise to a canonical isomorphism of cochain complexes $\Omega^\bullet(M) \to \mathfrak{X}^\bullet(M)$, where the first complex is given by the Nijenhuis differential on differential forms and the second by the Poisson–Lichnerowicz differential on multivector fields associated to $\pi^\sharp = n\omega^{\sharp-1}$, therefore to an isomorphism in cohomology. There are examples, in which the Nijenhuis cohomology of (M, N) can be computed easily, thanks to invariance under the action of Lie group G, as the invariant Nijenhuis cohomology $H^\bullet_G(M, N)$ of (M, N). Prof. Pedroni and collaborators conjectured conditions to have $H^\bullet_G(M, N) \simeq H^\bullet_{LP}(M, n\omega^{\sharp-1})$. We plan to investigate further this issue and extend the approach, to Poisson manifolds arising from symplectic Nijenhuis manifolds with Lie groupoid-algebroid symmetries.
- I gave a talk by the title "An gentle introduction to groupoids through examples"

References

[1] I. M. Gel′fand and I. J. Dorfman. Schouten bracket and Hamiltonian operators. *Funktsional. Anal. i Prilozhen.*, 14(3):71–74, 1980.

[2] Y. Kosmann-Schwarzbach. The Lie bialgebroid of a Poisson-Nijenhuis manifold. *Lett. Math. Phys.*, 38(4):421–428, 1996.

[3] F. Magri and C. Morosi. A geometric characterization of hamiltonian systems through the theory of Poisson-Nijenhuis manifolds. *Lett. Math. Phys.*, 38(4):421–428, 1996.

[4] I. Vaisman. Reduction of Poisson-Nijenhuis manifolds. *J. Geom. Phys.*, 19(1):90–98, 1996.

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