

## SHORT VISIT REPORT

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### Reduction of Poisson–Nijenhuis manifolds and integrability of bi-Hamiltonian systems

#### Grant Proposal.

Poisson - Nijenhuis manifolds were introduced by Magri and Morosi in [3] and constitute a geometric facet of Gel'fand and Dorfman's algebraic approach to integrable systems [1]. There exist a huge number of bi-Hamiltonian systems arising from Poisson - Nijenhuis manifolds. The characterization of Poisson - Nijenhuis manifolds in terms of Lie bialgebroids given by Kosmann - Schwarzbach in [2] allows to borrow Lie theoretic techniques to study the complete integrability of the associated dynamical systems; in particular, the study modular classes of Poisson manifolds and Lie algebroids has recently led to a better understanding of the geometry of some integrable hierarchies, tanks to the work of various authors.

In my recent work I used Kosmann -Schwarzbach's description to obtain a reduction result for Poisson - Nijenhuis manifolds under compatible actions as general as those of Poisson groupoids. Other reduction procedures have been studied earlier by Vaisman [4] and Magri et alia and represent a powerful method to the study of complete integrability.

The aim of my visit is to discuss with Prof. Magri and the members of his research group possible applications and to study examples of different reduction scenarios for Poisson-Nijenhuis manifolds, to extend my result to the case of holomorphic Poisson manifolds (a special case of Poisson - Nijenhuis manifolds) and to investigate the behavior of modular classes under reduction.

**Visit Report.** During my visit to Milan-Bicocca:

- The conditions for reducing Poisson-Nijenhuis manifolds under Poisson groupoid actions I earlier obtained were discussed in depth and simplified. Moreover, we were able to formulate a precise notion of a compatible action of a Lie bialgebroid on a Poisson–Nijenhuis manifold, the infinitesimal analog of a Poisson groupoid action.

- The problem of classifying linear and polynomial Poisson–Nijenhuis structures on dual of Lie algebras with (fixed KKS Poisson bivector) has been addressed. Beside of interest of its own – examples of compatible linear Nijenhuis tensors on dual of quadratic Lie algebras arise from solutions to the Yang Baxter equations – the study of this problem might produce concrete and interesting examples of Poisson–Nijenhuis manifolds on spaces of coadjoint orbits of Lie algebras, via my reduction procedure of the natural action of the cotangent prolongation groupoid.  
This might lead to a future collaboration with Prof. Magri and his collaborators.

### References

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- [4] I. Vaisman. Reduction of Poisson-Nijenhuis manifolds. *J. Geom. Phys.*, 19(1):90–98, 1996.

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