ESF-Short Visit Grant - Final Report

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The goal of the visit was to advance in some open questions in the theory of random matrices. Specifically, the research during the visit was supposed to be focused on (a) the universalities in the semiclassical analysis of hyperbolic systens of conservation laws, (b) the pole strubcture of *tronquué* solutions of Painlevé equations, (c) the asymptotic analysis of block Toeplitz determinants with applications to the statistical mechanics and quantum entanglement.

According to the agreement with his host, Dr. Its had been visiting SISSA from January 31 to February 17. During the visit, the following accomplishments have been made.

- The prospects of the use of the "universal String equation" (i.e. the recently found new spectral identity in the theory of 1D Schrodinger operator) in Dr. Grava's (SISSA) Riemann-Hilbert approach to Dubrovin's semiclassical asymptotics of the solutions of the KdV equation were discussed. The important similarities with the previous work of Dr. Its and P. Bleher on the asymptotic expansion of the partition function of the Hermitian matrix model were analyzed.

- The challenging problem of proving the complete absence of the poles in the relevant sectors for the *tronguué* Painlevé functions have been considered together with B. Dubrovin. A possible approach to the problem, based on the recently discovered possibility of the explicit evaluation of certain contour integrals of Painlevé functions, has been suggested.

- A new joint research project on the asymptotic evaluation of the block Toeplitz determinants has been launched together with Mattia Cafasso, SISSA. Specifically, the Toeplitz determinant corresponding to the so-called dimer model of statistical mechanics has been considered. The first and in fact the crucial step of the analysis, i.e. the relevant Wiener-Hopf factorization of the matrix symbol, was explicitly performed in terms of the Riemann theta functions.

- A joint project with Fabio Franchini from the Abdus Salam ICTP on the quantum entanglement in the XY spin chain has been continued. Indeed, the visit of Dr. Its to SISSA has produced an important breakthrough: the preliminary analysis of the zeta-function of the models' density matrix has allowed to describe the spectrum explicitly near its accomulation point. The analysis has also indicated that the eigenvalue multiplicities must have certain number theoretical meaning.

Two papers, based on the results obtained during the visit, are now in preparation. The one paper is based on the work done on block Toeplitz determinants while the other one is based on the results concerned with the density matrix of the XY chain.

The discussions on the first two topics above have build a solid background for launching in the future a joint project with Prof. Dubrovin and Dr. Grava on the universalities of the critical phenomena in nonlinear PDEs and related questions of the theory of *tronquué* solutions of Painlevé equations.

In addition, Dr. Its gave a short course on the asymptotic analysis of random matrices and orthogonal polynomials via the Riemann-Hilbert method. The title of the course was:

- *The Riemann-Hilbert approach in the theory of Toeplitz and Hankel determinants.*

The lectures were given at SISSA on Thursday, January 31, and on Thursday, February 7, 2008.