

SCIENTIFIC REPORT

JULY-SEPTEMBER 2008

Concerning the visit of Spyridon Kamvissis (Greece) to Gerald Teschl, Vienna, Austria, July-September 2008

1. PURPOSE OF THE VISIT

The purpose of the visit has been collaboration concerning the generalized Toda shock problem. In other words, we investigate the asymptotic behavior of the doubly infinite Toda lattice

$$(1) \quad \begin{aligned} \dot{b}_n &= 2(a_n^2 - a_{n-1}^2), \\ \dot{a}_n &= a_n(b_{n+1} - b_n), \\ n &\in \mathbb{Z}, \end{aligned}$$

with initial data approaching finite genus solutions at the two infinities.

2. DESCRIPTION OF THE WORK CARRIED OUT DURING THE VISIT

We have been able to write the relevant inverse scattering problem as a Riemann-Hilbert factorization problem on a hyperelliptic Riemann surface. We have generalised the g-function mechanism of Deift, Venakides, Zhou. The g-function is defined as the (multivalued) solution of a scalar problem on the Riemann surface. (Thus we avoid poles; the multivaluedness eventually disappears.) The defining condition at infinity gives some integral moment conditions that identify the "effective" spectrum that regulates the long time behavior. We have first focused on the case $g=1$ and we have found that even in that case it is more convenient to consider the factorization problem on a hyperelliptic Riemann surface.

3. DESCRIPTION OF THE MAIN RESULTS OBTAINED

The analysis of the generalized Toda shock reduces to the analysis of a Riemann-Hilbert factorization problem on a hyperelliptic Riemann surface. In the case of genus 1 the asymptotic analysis of the Riemann-Hilbert factorization problem can be effected via the 1-gap g -function mechanism (generalized to a Riemann surface). This extends the generalization of the Deift/Zhou theory that began in the previous papers by the authors (also supported by ESF), in particular arXiv:nlin/0607053, arXiv:0705.0346 and arXiv:0805.3847. The generalization to higher genus should not be hard.

4. PROJECTED PUBLICATIONS/ARTICLES RESULTING FROM OUR GRANT

Not clear yet.

5. FUTURE COLLABORATION WITH HOST INSTITUTION

A further visit is planned for 2009.

Spyridon Kamvissis

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