

1.1 A short description of the proposed project work (about 250 words)

Title: “On the elliptic Poisson tensors and related integrable systems”

During the nineties Odesskii and Feigin introduce the notion of elliptic Poisson tensors q_n as the classical counterpart of the “quantized” elliptic algebras Q_n generalizing, to every dimension n , the 4 dimensional Sklyanin algebra (see e.g. [2]). The ingredients to construct Q_n are an elliptic curve \mathcal{E} and a point η . The structure coefficients of the algebra depend rationally on the theta functions related to \mathcal{E} . The point η play the role of quantum deformation parameter.

It is always possible to construct some finite dimensional integrable systems Hamiltonian with respect to q_n . The approach for the construction of the commuting Hamiltonian flows are essentially two: in [4, 5] the authors find, using purely algebro-geometric methods and only for even dimensions, an abelian subalgebra of q_n which allows the construction of integrable commuting flows; in [3] the author change his point of view: under a suitable change of variables, based on the relation between θ functions and Weierstrass \mathcal{P} functions on the torus, the q_n tensors become a pencil of three compatible Poisson structures. The role of pencil parameters is played by the g_2 and g_3 constants appearing in the elliptic curve related to the torus. By means of the standard bi-Hamiltonian methods involving the Casimir of the pencil it is possible to construct integrable systems.

In this context, with prof. Rubtsov and his PhD student Tagne Pelap interested in Sklyanin algebras [8], we have studied of the extension of the class of q_n tensors given by quadratic Poisson tensors invariant with respect to the discrete Heisenberg group (H). We have classified all the tensors of this class until the dimension 6 and, for every dimension, we show that every quadratic H-invariant Poisson tensors (in particular of the elliptic ones) are unimodular. Next we concentrate our study on the dimension 5 and we show that the two inequivalent elliptic structures $q_{5,1}$ and $q_{5,2}$ are related by a (birational) Cremona transformation.

We are now studying if the unimodularity property plays some role on the structure of the possibly related integrable systems. Moreover, inspired by [3], during a visit at the Bicocca University of Tagne Pelap, we start, with prof. M. Pedroni, the study of the hyperelliptic generalization of q_n .

1.2 Aim of the visit

My personal and fruitful collaboration with prof. Rubtsov is started three years ago (with Prof. M. Pedroni) with the study of nonlocal Poisson structures of Camassa-Holm hierarchy [6].

During the two weeks of my visit we wish to continue our study of elliptic Poisson tensors started in 01/2008 (MISGAM exchange grant no. 1788). We would finish our paper [7] and then we would study the possible hyperelliptic or rational extensions of the polynomial q_n Poisson tensors and the possible relations with the cubic pencils given in [1]. My period in Angers is really important in order to continue our collaboration in the domain of elliptic integrable systems.

References

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