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Antonio Moro

Academic Degrees

October 2004: **PhD degree in Physics,** University of Lecce (now University of Salento), Thesis: "*Integrable Structures in Nonlinear Geometrical Optics and Quasiclassical Dbar-dressing method".*

December 2000: **Degree in Physics,** 110/110 cum laude, University of Lecce (now University of Salento), Thesis: *"Simulation of gauge theories in Hamiltonian formulation".*

July 1996: **Maturita' Scientifica (A level),** final mark 60/60, Liceo Scientifico Galileo Ferraris, Taranto (Italy).

Career/ Employment

April 01, 2006 – Up to now
Research Associate at the Department of Mathematical Sciences, Loughborough University (UK). Duration of the contract: 3 years.

February 01, 2005 – March 2006.
Post-Doc fellow (Assegno di Ricerca) at Department of Physics, Lecce University (Italy). Topic: *"Dispersionless systems and nonlinear optical phenomena".*

June 2001 – June, 2004
Physics PhD student at Department of Physics, Lecce University (Italy).

1996 – 2000
Physics undergraduate student at University of Lecce (Italy).

Selected Publications

1) FERAPONTOV E V, MORO A, SOKOLOV V V. 2008. Hamiltonian systems of hydrodynamic type in 2+1 dimensions. accepted for publication in *Communications in Mathematical Physics.* Preprint arXiv:0710.2012 (2007)
2) MORO A AND KONOPELCHENKO B. 2006. High frequency integrable regimes in nonlocal nonlinear optics. *Journal of Geometry and Symmetry in Physics*, **7**, pp. 37-83
3) MORO A. 2006. On the nonlocal nonlinear Schrodinger equation and its integrable regimes. E. BARLETTA (eds). *Lecture Notes of SIM vol.5* . Potenza: Dipartimento di Matematica, Universita' della Basilicata, pp. 173-197
4) KONOPELCHENKO B AND MORO A. 2004. Integrable equations in nonlinear geometrical optics. *Studies in Applied Mathematics,* **113,**

pp. 325-352

5) BOGDANOV L, KONOPELCHENKO B AND MORO A. 2006. Symmetry constraints for real dispersionless Veselov-Novikov equation. *Journal of Mathematical Science,.* **136**(6), pp. 4411-4418

6) KONOPELCHENKO B AND MORO A. 2004. Geometrical optics in nonlinear media and integrable equations*. Journal of Physics A: Mathematical and General,* **37,** pp**.** L105-L111

Invited Seminars

- March 11, 2008, Department of Mathematics and Statistics, Glasgow University, UK. Title: "*Hamiltonian Systems of hydrodynamic type in 2+1 dimensions".*
- October 24, 2007, Department of Applied Mathematics, State University of Campinas, Campinas, Brazil. Title: *Nonlinear Geometric Optics: Vortices and Nonlocality*.
- October 22, 2007, Departamento de Matematica, Universidade de Sao Paulo, San Carlos, Brazil. Title: "*Hamiltonian systems of hydrodynamic type in 2+1 dimensions*".
- May 09, 2007, Department of Applied Mathematics, Liverpool University (UK). Title: '' *Nonlinear Geometric optics, vortices and nonlocality.*"

Selected Conferences

1. September 25-27, 2007, Milano (Italy), MISGAM Conference on Integrable Systems. **Invited Talk** – "*Hamiltonian systems of hydrodynamic type in 2+1 dimensions*".
2. September 7-12, 2006, Colmenarejo (Madrid, Spain). Satellite Conference of the International Conference of Mathematicians (Madrid August 22-30): Integrable Systems in Applied Mathematics.
   Invited Talk -- "*Nonlinear Geometric Optics, Compressible Flows and Nonlocal Perturbations*" .

3. June 22 – July 1, 2006, Gallipoli (Italy), Nonlinear Physics. Theory and experiment.IV
   Talk – "*High frequency integrable regimes in nonlocal nonlinear optics*".

4. June 24 – July 03, 2004 Gallipoli (Lecce), Workshop - Nonlinear Physics: Theory and experiment, **Talk –** "*Dispersionless Veselov-Novikov equation and integrable nonlinear geometrical optics".*

CASE FOR SUPPORT

Short Visit Grant

Critical behaviour of the focusing nonlinear Schrödinger equation and nonlocal perturbations

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1. PRELIMINARY

Nonlocality arises importantly in many areas of physics as, for example, a manifestation of transport mechanisms, long range forces or many body interactions. In particular, in nonlinear optics, nonlocal effects occur in the propagation of light beams in photorefractive media, thermal nonlinear media, liquid crystals.[1–4]

It was observed that a suitable nonlinear nonlocal response of the medium can prevent the catastrophic collapses of self-focused light beams[2,6] Moreover, there are experimental evidences that nonlocal media can support the propagation of new soliton-like light beams.[1] Recently, Ghofranhia et al.[2] have studied, for a particular model, how nonlocality affects the shock wave formation in the low dispersion limit. It turns out that shocks survives with nonlocality and, despite the local case, shock dynamics prevails over modulational instability.

A paraxial light beam propagating in a nonlinear nonlocal medium is described by the following nonlocal nonlinear Schrödinger type (NNLS) equation of the form

$$i\epsilon\psi_t + \frac{\epsilon^2}{2}\psi_{xx} + \eta(|\psi|^2)\psi = 0, \tag{1}$$

where ψ is the scalar component of a linearly polarized light beam and ϵ is a small parameter ad η the nonlocal nonlinear potential. In this physical set up the variable t is associated with the propagating direction of the beam and it is not the "physical time". Introducing the slow variables

$$u = |\psi|^2, \qquad v = \frac{\epsilon}{2i}\left(\frac{\psi_x}{\psi} - \frac{\psi_x^*}{\psi^*}\right)$$

NNLS equation can be recast in the following form

$$\begin{aligned} &u_t + (uv)_t = 0 \\ &v_t + vv_x - \eta_x + \epsilon^2\left(\frac{1}{2}\frac{u_x^2}{u^2} - \frac{u_x x}{u}\right)_x = 0. \end{aligned} \tag{2}$$

Nonlocality η is specified via an integral expression of the form

$$\eta = \int_{-\infty}^{\infty} R\left(\frac{x}{\epsilon}, \frac{y}{\epsilon}, \sigma\right) u(y) d\left(\frac{y}{\epsilon}\right)$$

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where the distribution $R(x,y,\sigma;\epsilon)$ depends on a parameter σ which is the typical range on non-locality. Note that the nonlocality can be also given by a certain non-homogeneous differential equation of the form

$$L(\eta,\eta_x,\eta_{xx},\dots)=u.$$

According to the model under study the nonlocality can contribute to the dispersionless limit ($\epsilon\to 0$) with dissipative and also dispersive corrections. As an axample, consider a Gaussian nonlocal response of the form

$$\eta=\int_{-\infty}^{+\infty}\frac{1}{\sqrt{\pi}\sigma}e^{-\frac{(x-y)^2}{\epsilon^2\sigma^2}}u(y)d\left(\frac{y}{\epsilon}\right).$$

We immediately see that if the range of nonlocality σ is fixed there are no nonlocal contributions in the dispersionless limit

$$\lim_{\epsilon\to 0}\int_{-\infty}^{+\infty}\frac{1}{\sqrt{\pi}\sigma}e^{-\frac{(x-y)^2}{\epsilon^2\sigma^2}}u(y)d\left(\frac{y}{\epsilon}\right)=\delta(x-y).$$

Only very high nonlocalities of typical range $\sigma=\alpha/\epsilon$ survive in the dispersionless limit. Thus, for finite range nonlocalities we can adopt a kernel expansion of the form

$$R(x,y,\sigma)=\delta(x-y)+\epsilon\sigma R_1(x,y)+\epsilon^2\sigma^2R_2(x,y)+\dots$$

where $R_i(x,y)$'s describe a set of nonlocal perturbations. The case $R_{2n+1}=0$ and $R_{2n}=\delta^{(2n)}(x-y)$ has been recently studied by Ghofraniha *et al.* [2] who provided also an interesting experimental feedback of their theoretical and numerical analysis. Their system looks like as follow

$$u_t+(uv)_x=0,\qquad v_t+vv_x-\eta_x=0,\qquad \eta=u+\epsilon^2\sigma^2\eta_{xx}. \tag{3}$$

Formally, the equation for η is equivalent to the following expansion

$$\eta=u+\epsilon^2\sigma^2u_{xx}+\epsilon^4\sigma^4u_{xxxx}+\dots=\int_{\infty}^{+\infty}\left(\delta(x-y)+\epsilon^2\sigma^2\delta''(x-y)+\dots\right)u(y)dy$$

and, obviously, σ plays the role of the range (or degree) of nonlocality. In the mentioned paper it is shown that higher nonlocality favours shock dynamics over filamentation.

2. PROGRAMME

Recently, Dubrovin-Grava-Klein have studied the critical behaviour of the NLS equation in connection with the *tritronquée* solution to the Painlevé I equation. They have also conjectured that the behaviour of the solutions at the critical point is universal, i.e. it does not depend on the initial conditions. Their approach is based on two observations: first there is an unique ϵ-power series extension of any first integral of dispersionless NLS equation which commutes with the NLS Hamiltonian; second, given such an extension, any solution to the NLS equation in the class of the power series is obtained from the equations

$$x=vt+\frac{\delta H_f}{\delta u(x)},\qquad s=ut+\frac{\delta H_f}{\delta v(x)}$$

where H_f is the extension associated with the first integral f of the dispersionless NLS which commutes with the Hamiltonian H of the full NLS, that is

$$\{H, H_f\} = 0.$$

The Poisson bracket above is defined as follows

$$\{u(x), v(y)\} = \delta'(x-y), \qquad \{u(x), u(y)\} = \{v(x), v(y)\} = 0. \tag{4}$$

The aim of the present project is to investigate the possible extension of their approach to NNLS-type equation, in order to give a rigorous description of the nonlocal effects at the critical point. The first step is the analysis of the system (3). In particular, note that it can be written equivalently as follows

$$u_t = \{u, H\}, \qquad v_t = \{v, H\}$$

where the Hamiltonian is $H = \int (uv^2 - u\eta)/2dx$.

The interplay between standard dispersive corrections and nonlocal ones in the shock dynamics will be also investigated. A deep understanding of this aspect in 1+1-dimensions is the base for a future analysis of the 2+1-dimensional case where the role of nonlocality is even more interesting. In 2+1 dimensions the standard NLS equation exhibits wave collapse in finite time. In other word the dispersive term is not sufficient to hamper the break-down as it happens 1+1-dimensions. A number of numerical and experimental results suggest that nonlocality have a "smoothing" effect and eventually wipes the singularity out.

3. AIM OF THE VISIT.

I have been interested in the nonlocal nonlinear optics for few years. In particular, my contribution in this field is the introduction of a class of nonlocal perturbations in nonlinear geometric optics modelled via the integrable hierarchies of dispersionless PEDs.[7,8]

The recent studies on the Hamiltonian perturbations to disperionless PDEs by Dubrovin, Liu, Zhang[9,10] and in particular the critical point analysis for the NLS equation proposed by Dubrovin, Grava and Klein[11] offer a new approach for investigating the role of nonlocality. Visiting Proffs Boris Dubrovin and Tamara Grava at the SISSA in Trieste is crucial to achieve the purposes of the present project. I am confident that a two weeks visit will be enough to

- get familiar with their method;
- extend the method to a new class class of problems (nonlocal models in nonlinear optics) as outlined above;
- start a promising collaboration.

Best period for the visit is from 30th May to 12th June 2008.

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2. Neda Ghofraniha, Claudio Conti, Giancarlo Ruocco, Stefano Trillo, "Shock in nonlocal media", *Phys. Rev. Lett.*, **99**, pp. 043903 1-4, 2007.
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8. B. Konopelchenko and A. Moro, "Integrable equations in nonlinear geometrical optics", *Stud. Appl. Math.*, **113**, pp. 325-352, 2004.
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10. B. Dubrovin, "On Hamiltonian perturbations of hyperbolic systems of conservation laws, 2: universality of critical behaviour", *Comm. Math. Phys.*, **267(1)**, pp. 117-139, 2006.
11. B. Dubrovin, T. Grava and C. Klein, On universality of critical behaviour in the focusing nonlinear Schrödinger equation, elliptic umbilic catastrophe and the *tritronquée* solution to the Painlevé I equation, `arXiv:0704.0501` (2007).