Project work and aim of the visit

The main subject of my research is the class of integrable partial differential equations known as integrable disperionless hierarchies. This field has played a key role in many aspects of mathematical physics, such as gas dynamics, shallow water theory and nonlinear elasticity; for example, many of these equations can be derived as dispersionless limits of fully integrable systems, while others enter in the Whitham averaging description. In recent years, important relations between integrable hierarchies and geometric structures have been established; for instance we recall the connections with Frobenius manifolds and with deformations of family of conformal maps on complex domains, given by Loewner evolution or Laplacian growth.

The main objects in dispersionless integrable hierarchies are given by the so-called *systems of hydrodynamic type.* These, in the case of $(1+1)$ dimensions, are systems of n quasilinear partial differential equations in the variables $u^1(x,t), \ldots, u^n(x,t)$, of the form

$$\frac{\partial u^i}{\partial t} = \sum_{j=1}^{n} V^i_j(u^1, \ldots, u^n) \frac{\partial u^j}{\partial x}, \qquad i = 1, \ldots, n, \tag{1}$$

where suitable boundary conditions are usually specified. It is well known that the functions u^i can be interpreted as coordinates on a differentiable manifold M of dimension n; moreover, many properties of the system of PDEs can be deduced by studying the differential geometry of M. An important example of this relation is given by the *Poisson brackets of hydrodynamic type*, which can be used to describe an Hamiltonian formalism for hydrodynamic type systems. These Poisson structures have been introduced by Dubrovin and Novikov in the local case, and by Mokhov and Ferapontov in the nonlocal one. The quantities defining a Poisson bracket can be expressed in terms of a pseudo–metric, the corresponding Christoffel symbols and Riemann curvature tensor on the manifold M. Moreover, it is important to recall that the concept of integrability of a sysytem of type (1), developed by Tsarev, is also related with the vanishig of suitable tensorial quantities, such as the Haantjes tensor and the semi–Hamiltonian tensor.

In the last few years a lot of interest in the research has been devoted to the study of the so called chains of hydrodynamic type. These are natural generalizations of hydrodynamic type systems, in the case that the variables u^i are infinitely many and the V^i_j suitably sparse. This kind of generalization is not merely abstract, as many important equations of mathematical physics can be described through this formalism. Among these, we recall the disperionless limit of the Kadomtsev- Petviashvili equation (also known as Benney chain) or the dispersionless two dimensional Toda lattice.

During my PhD, I have been working on different problems related with hydrodynamic chains. In one of these, together with my supervisor Dr. John Gibbons and in collaboration with Dr. Paolo Lorenzoni, we determined the Hamiltonian structures of all reductions of the Benney chain, of which only a few examples were known to be Hamiltonian before. More specifically, from the well known relation between Benney reduction and families of conformal maps (solution of a system of Loewner equations), we wrote explicit formulae for the Poisson tensors of the reductions in terms of the conformal maps. These Hamiltonian structures are in general nonlocal, of Ferapontov type.

A second problem, developed again together with Dr. Gibbons and Dr. Lorenzoni, has been devoted to the study of purely nonlocal Poisson structures for systems of hydrodynamic type. These can be constructed provided there exists a set of commuting flows satisfying a sort of orthogonality condition; the resulting structures complete the picture of the Poisson brackets of hydrodynamic type, together with the Dubrovin-Novikov local operators and the brackets found by Ferapontov and Mokhov, where the operator is given by a local part together with a nonlocal one.

As a natural consequence of the study of Hamiltonian structures for semi-Hamiltonian systems, the aim of the project is to begin a description of dispersive deformations for such systems. This problem has been studied by Dubrovin and Zhang in the particular case when the hierarchy admits two compatible local structures. An important result in this direction has been proved by Getzler and independently by Degiovanni, Magri and Sciacca; they proved that the Poisson cohomology for a local Hamiltonian structure of hydrodynamic type is trivial. Since a semi-Hamiltonian system does not admit in general any local structure, but only nonlocal ones, the first step in the dispersive deformation of semi-Hamiltonian systems is the study of the Poisson cohomology for nonlocal Hamiltonian structures of hydrodynamic type. The aim of the visit is to continue the collaboration with Dr. Lorenzoni, and to discuss some points of the above project with the members of the Mathematical Physics Group of the Department, namely professor Gregorio Falqui and professor Franco Magri.