

# Research project

## Quasilinear PDEs and conformal structures in projective space

I plan to investigate second order quasilinear equations of the form

$$f_{ij}u_{x_i x_j} = 0$$

where  $u$  is a function of  $n$  independent variables  $x_1, \dots, x_n$ , and the coefficients  $f_{ij}$  are functions of the first order derivatives  $p^1 = u_{x_1}, \dots, p^n = u_{x_n}$  only. The natural equivalence group of the problem is isomorphic to  $SL(n+1, R)$ , which acts by projective transformations on the space  $P^n$  with coordinates  $p^1, \dots, p^n$ . The coefficient matrix  $f_{ij}$  defines on  $P^n$  a conformal structure  $f_{ij}(\mathbf{p})dp^i dp^j$ . I intend to concentrate on the case  $n = 3$ . The necessary and sufficient conditions for the integrability of such equations by the method of hydrodynamic reductions were derived in our previous work with Burovskiy and Tsarev. These conditions constitute a complicated over-determined system of PDEs for the coefficients  $f_{ij}$ , which is in involution. It was demonstrated that the moduli space of integrable equations is 20-dimensional. Based on these results, I plan to show that any equation satisfying the integrability conditions is necessarily conservative, and possesses a dispersionless Lax pair. Reformulated in differential-geometric terms, the integrability conditions imply that the conformal structure  $f_{ij}(\mathbf{p})dp^i dp^j$  is conformally flat, and possesses an infinity of 3-conjugate coordinate systems of null curves. I plan to obtain a compact tensor representation of the integrability conditions as differential-geometric constraints connecting a projectively flat connection and a conformally flat structure.

I also plan to give a short course of lectures on my recent research.

## CV of Eugene Ferapontov

**Name:** Eugene V Ferapontov

**Date of Birth:** 3 August 1960

**Marital status:** married, 2 children

**Education:** Faculty of Mechanics and Mathematics, Lomonosov  
Moscow State University:  
1976–1982 student  
1982–1985 PhD student

**Qualifications:** PhD Thesis (1987):  
‘Bäcklund transformations of quasilinear systems of first  
order partial differential equations’, Moscow State Uni-  
versity, Faculty of Mechanics and Mathematics, Depart-  
ment of Differential Geometry and Topology. Supervisor:  
Professor A M Vassiliev.

**Publications:** Over 70 research papers on differential geometry and in-  
tegrable systems in refereed academic journals. Chapters  
in books and conference proceedings.

**Position:** Reader in Differential Geometry, Department of Mathe-  
matical Sciences, Loughborough University.

### Recent conference talks:

Methods of Integrable Systems in Geometry, Durham, UK, August 2006.

Conference NEEDS-07, Barcelona, Spain, June 2007.

ENIGMA Conference on Mathematical Physics, KTH, Stockholm, June 2007.

Island-3 meeting on Algebraic Aspects of Integrable Systems, Islay, Scotland, July  
2007.

Odense Winter School, Odense, Denmark, January 2008.

## Selected publications

- [1] E. V. Ferapontov and K. R. Khusnutdinova, On integrability of (2+1)-dimensional quasilinear systems, *Comm. Math. Phys.* **248** (2004) 187-206.
- [2] E. V. Ferapontov and K. R. Khusnutdinova, The characterization of 2-component (2+1)-dimensional integrable systems of hydrodynamic type, *J. Phys. A: Math. Gen.* **37**, no. 8 (2004) 2949 - 2963.
- [3] E. V. Ferapontov and K. R. Khusnutdinova, Hydrodynamic reductions of multi-dimensional dispersionless PDEs: the test for integrability, *J. Math. Phys.* **45**, no. 6 (2004) 2365-2377.
- [4] E. V. Ferapontov, K. R. Khusnutdinova and M. V. Pavlov, Classification of integrable (2+1)-dimensional quasilinear hierarchies, *Theoret. Math. Phys.* **144**, no. 1 (2005) 907-915.
- [5] E. V. Ferapontov, K. R. Khusnutdinova and S. P. Tsarev, On a class of three-dimensional integrable Lagrangians, *Comm. Math. Phys.* **261**, no. 1 (2006) 225-243.
- [6] E. V. Ferapontov and K. R. Khusnutdinova, Double waves in multi-dimensional systems of hydrodynamic type: the necessary condition for integrability, *Proc. Royal Soc. A* **462** (2006) 1197-1219.
- [7] E. V. Ferapontov, K. R. Khusnutdinova, D. G. Marshall and M. V. Pavlov, Classification of integrable Hamiltonian hydrodynamic chains associated with Kupershmidt's brackets, *J. Math. Phys.* **47**, no. 1 (2006) 13 pp.
- [8] E. V. Ferapontov and D. G. Marshall, Differential-geometric approach to the integrability of hydrodynamic chains: the Haantjes tensor, *Math. Ann.* **339**, no. 1 (2007) 61-99.
- [9] E. V. Ferapontov, L. Hadjikos and K.R. Khusnutdinova, Integrable equations of the dispersionless Hirota type and hypersurfaces in the Lagrangian Grassmannian, arXiv: 0705.1774 (2007), submitted to *Proc. London Math. Soc.*
- [10] E. V. Ferapontov and A. V. Odesskii, Integrable Lagrangians and modular forms, arXiv:0707.3433 (2007).