Theta-functional approach to the theory of non-abelian monopoles

Proposal for MISGAM grant application

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The construction of families of explicit solutions to the monopole (Bogomolny) equations in the case of gauge group $su(2)$ and higher charge ($n \geq 2$) remains an important and unsolved problem. Hitchin developed an algebro-geometric approach to the problem based on the ADMHN (Atiayh-Drinfeld-Hitchin-Manin-Nahm) construction. Here each monopole solution of the theory is built from a compact algebraic curve whose moduli are constrained by a number of non-trivial constraints, the so-called Hitchin constraints. At the present time only a few non-axially symmetric monopole curves are known: these possess Platonic symmetry and do not admit the theoretically possible number of free parameters.

In recent publications [1]-[3] H.W. Braden (host) and the applicant have obtained a number of results relevant for reconstructing the gauge and Higgs potential of the underlying field theory. These are based on theta-functions appropriate to the monopole curve. The implementation of these functions involves routine (but technically complicated) data requiring a detailed knowledge of the curve (its moduli, periods, homology basis etc.). The aforementioned problem of finding new monopole curves beyond those with Platonic symmetry however remains unsolved though strong partial results have been obtained.

In searching for new monopole curves we concentrated on the first non-trivial case, that of charge three, where the curve of genus four was assumed of the form $y^3 = (x - a_1) \dots (x - a_6)$. Using modern and old investigations of this curve we developed analytic and numeric machinery that seems sufficient to solve the problem in this particular case. At the moment we conjecture that: *There are no charge three monopoles associated these curves beyond that with tetrahedral (Platonic) symmetry.*

The applicant's visit to Edinburgh University aims to complete our investigations of this problem: either to prove the conjecture or to find a counter example (which would be even more impressive). To this end we have developed several new tools, but to complete our task the author's need time together.

In our unpublished work we recently found that our analysis can be extended C_3 cyclic monopoles

$$w^3 + \alpha w z^2 + \beta z^6 + \gamma z^3 - \beta = 0$$

and we believe this family of curves contains new monopole curve.This latter analysis involves Braden's graduate student A.D'Avanzo. He also has another PhD student, Timothy Northover working on related matters of finite-gap integration who will benefit from a visit.

References

[1] Enolski, Victor and Braden, Harry. Finite-gap integration of the $SU(2)$ Bogomolny equations. *Glasgow Math. J.* 2008, *in press* arXiv: math-ph/0806.1807

[2] Braden H.W. and Enolski V.Z. Monopoles, Curves and Ramanujan Lect. Notes Math. 2007 in: Proceedings of the Workshop: Riemann Surfaces- Analytical and Numerical Methods, May 31 -June 02, 2005, Max Planck Institute, Leipzig, ed. A.Bobenko and Ch. Klein arXiv: math-ph/0704.3939.

[3] Braden H.W and Enolski V.Z., Remarks on the complex geometry of the 3-monopole. arXiv: math-ph/0601040, 2006.