

Scientific Report on the project

Nonlocal Poisson structures for the Camassa-Holm hierarchy

I have spent a week in Milan in June 2007 (arrival date: june 10- departure june 16). My host was Prof. Franco Magri and my principal collaborators in this Project were Prof. Marco and Dr. Giovanni Ortenzi. I have also some scientific contacts with Prof. Gregorio Falqui and Dr. Paolo Lorenzoni. I had given a talk on the Department Seminar, Friday, june 15 about Generalized Geometry and some models in Geophysical Dynamics.

Apart of the main Project subject I have had a long discussion with Profs. Magri, Pedroni and other participants of Bicocca Seminar about different approaches to the Ruijsenaars duality in many-body integrable systems.

The goal of the Project is to give a description to all “higher” (non-local) Poisson structures for the Camassa-Holm and some related hierarchies.

We have started to construct an analogue of the generating series (known previously in the KdV case) for these “higher” Poisson structures of CH during the visit of G. Ortenzi and M. Pedroni in Angers in November 2006.

The main aim of my short visit to the University of Milan -Bicocca was to update and finish our paper which we have started in November 2006.

During this week we have updated a draft of our paper with M. Pedroni and G. Ortenzi and have found some new directions to continue our research. Our discussions were focused on the following questions: what kind of non-locality admits the CH hierarchy respectively to KdV? Does its “symplectic” counterpart (all “higher” symplectic forms) has the same type of non-locality? We have realised that(in contrary to KdV) the CH hierarchy admits a version of a “strong” non-locality. We have also understood how to extend our computations to the case of some other tri-hamiltonian systems. Main example of our new computations was the Ito hierarchy (two-component system which is a partial example of Antonovicz - Fordy polynomial hierarchies). We have used the coresponding linear problem and we have wrote the matrix analogues of our basic relations for the genrating series of higher Hamilton structures.

We have discussed also a group-theoretic approach to the higher CH Hamilton structures and we are going to study it in our future publications. It was remarked in my paper with A. Gorsky, A. Marshakov and A. Orlov that there is an interesting link between the 3-rd non-local KdV structure and the symplectic structure of Sl_2 -quantum gravity in the upper-triangular coordinate F (with is the “Schwarz potential“ for the KdV: $u Sch(F)$). Then, if the third non-local KdV Poisson bracket for u reads as

$$\{F(x), F(x)\} = F'(x)F'(y)\epsilon(x - y)$$

(the bracket which looks like the ”Sokolov bracket“ to Krichever-Novikov equation). In other hand, we have shown that this bracket can be written as an r -matrix bracket

$$\{F(x), F(x)\} = [r(x - y), F(x) \otimes F(y)]$$

with $r(x - y) = \epsilon(x - y)\partial_x \otimes \partial_y$ which is, again, stress the Poisson-Lie group nature of the third KdV brackets.

We are sure that similar questions for the CH hierarchy as well as for some close tri-hamiltonian two-component systems (like the Ito and the Caup-Kupershmidt) deserve to

be explored and this is one of goals of our future collaboration. We hope to relate this higher structures with the famous triple of Lie algebra cocycles of Arbarello-de Concini-Kac.

We are planning to finish our updated paper this summer (2007) and to submit it to one of special Journals (Physics Letters A, Journal of Physics A or Nonlinearity).

We are planning a long visit of G. Ortenzi in Angers in near future as well as my participation in a one-week Workshop in Milan (September 2007).

Professor V. Roubtsov
Angers, 20.06. 2007