Report for a short visit MISGAM grant

Applicant: Tamara Grava
Scuola Internazionale Superiore di Studi Avanzati, (SISSA)
via Beirut 2-4
34014 Trieste
ITALY
phone: +390403787445
email: grava@sissa.it
web-page: http://people.sissa.it/grava
Host: Bertrand Eynard
Service de Physique Théorique
Orme des Merisiers
CEA Saclay
F-91191 Gif-sur-Yvette Cedex

Scientific report: large N limit of random matrix model in the multi-cut case.

The topics discussed during the visit are the following.

- In [EO] Eynard and Orantin define, for any arbitrary algebraic curve $E(x, y) = 0$, an infinite sequence of complex numbers $F^{(g)}(E)$, computed as residues of meromorphic forms on the curve. Out of these $F^g(E)$, they build a formal power series:

$$\log Z_N(E) = \sum_{g=0}^{\infty} N^{2-2g} F^{(g)}(E),$$

  and show that Z_N satisfies Hirota bilinear relation and it is a formal τ function associated to the curve.

  From any algebraic curve $E(x, y)$, with an assigned meromorphic function x, Dubrovin has defined a Frobenius manifold [D]. For any Frobenius manifolds Dubrovin and Zhang [DZ] constructed the generalized loop equations for some some functions $\{F_g\}_{g\geq 0}$ and were able to solve them for $g \leq 2$. In this project we have started to investigate whether the functions F_g in [DZ] and $F^{(g)}$ in [EO] coincides.

- We consider partition function of a random matrix model,

$$Z_N = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \prod_{1\le j<k\le N} (\xi_j - \xi_k)^2 e^{-N\sum_{j=1}^N V(\xi_j)} d\xi_1 \dots d\xi_N = \tag{1}$$

where $V(\xi)$ is a polynomial,

$$V(\xi) = \sum_{j=1}^{2d} t_j \xi^j, \quad t_{2d} > 0. \tag{2}$$

The asymptotic expansion of Z_N as $N \to \infty$ in the so called multi-gap case has been obtained in [BDE] using a saddle point argument and takes the form

$$Z_N \simeq e^{-N^2 F_0} \theta\left(\frac{N\boldsymbol{\Omega}}{2\pi}; B\right). \tag{3}$$

In the above formula $\theta(\boldsymbol{z}; B) = \sum_{\boldsymbol{n}\in\mathbb{C}^g} e^{\pi i <\boldsymbol{n}, B\boldsymbol{n}> + 2\pi i <\boldsymbol{z},\boldsymbol{n}>}$ is the Riemann θ-function defined on the Jacobi variety of the hyperelliptic Riemann surface $y^2 = \prod_{k=1}^{2g+2}(\xi - u_k)$, B is the period matrix with respect to a canonical basis of holomorphic differentials and the vectors $\boldsymbol{\Omega} = (\Omega_1, \Omega_2, \dots, \Omega_g)$ are given by the relation

$$\Omega_j = 2\pi \int_{u_{2j+1}}^{u_{2g+2}} \psi(\xi) d\xi.$$

The formula (3) represents the leading order expansion of Z_N. Higher order corrections have been obtained by [E]. We started to investigate whether such corrections can be obtained from a Riemann-Hilbert approach and Deift/Zhou steepest descent method.

Bibliography

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