Visit of Giovanni Ortenzi to prof. Vladimir Rubtsov, Dept. Math. LAREMA- Univ. Angers (France)

Purpose of the visit

The purpose of the visit is the study of the two different notions of duality between integrable systems present in literature for the finite and the infinite dimensional case (see the proposed project work).

Description of the work carried out during the visit and main results obtained.

- **Elliptic Poisson tensors** Odesskii and Feigin (see e.g. [1]) generalize to arbitrary dimensions the 3 dimensional Artin-Tits and the 4 dimensional Sklyanin elliptic algebras constructed from a given elliptic curve $\mathcal{E}$ and a point η on $\mathcal{E}$. In a standard way, it is possible to associate the corresponding Poisson algebras, which are, in some sense, the classical limit in the parameter η of the quantistic commutator related to the elliptic algebras.
  We have observed that all the Poisson tensors constructed in such a way admit as Poisson morphisms a pair of linear change of variables generating the finite Heisenberg group of suitable dimensions. We have classified all the Heisenberg invariant quadratic Poisson tensors until the dimensions 6. Next we focus our attention on the dimension 5. The interest of this particular case is twofold: It is the first case when there is two different Poisson tensors generated by the Odesskii-Feigin construction and, being odd dimensional, it is not known how to relate to this algebra an integrable system [1]. We find a bi-rational "change of variables", the Cremona transformation, relating the two 5 dimensional Poisson tensors. We conjecture that this relation could be interpreted as a duality relation between integrable systems but, at the moment, we do not yet solved the problem of integrable system for the odd dimensional elliptic Poisson tensors.

- **The study of the Camassa-Holm equation** We have continued the study of the bi-Hamiltonian dual couple of systems Camassa-Holm/Korteweg-de Vries started in 11/2006 with the recently published paper [3]. Our attention has been focused on the analogue of the Ur-KdV equation for CH. This last equation, introduced in [4], shows, among many other properties, the existence of a couple of local symplectic structures related to the KdV hierarchy. We try to extend this result to the CH with zero critical velocity, but we do not yet have a good analogue. However from many interesting discussion we obtain some side result on the recurrence operator of the KdV hierarchy, whose analogue for the CH case could be useful in order to answer our question.

Future collaboration with host institution

We plan to pursue the collaboration with prof. Rubtsov. First of all we have to write the paper on the elliptic Poisson structures. Moreover we want to understand if it is possible to associate some integrable systems to the odd-dimensional elliptic Poisson tensors. In the particular case of dimension 5 we want to understand if the Cremona transformation generates some duality between the maybe related integrable systems .
On the side of infinite dimensional systems, we plan to study, in collaboration of prof. M. Pedroni, the quasi-Hamiltonian formalism of the critical CH equation.

Projected articles resulting or to result from your grant

G. Ortenzi, S. Pelap, V. Rubtsov *On the low dimensional elliptic Poisson structures and the discrete Heisenberg invariance*, draft

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[1]Actually only for the even dimensional case in [2] the authors associate an integrable systems to the elliptic algebras.

Other remarks

During my period in the Angers Mathematics Department I participate to the mathematical-physics seminars a I have given lecture on the subject of bi-Hamiltonian geometry, on the relation with the linear evolution on the Sato Grassmannian, and on the reciprocal transformations.
I have had many interesting and stimulating interactions with the mathematical-physics group of the Department. In particular I discussed with the PhD student J. Dongo on the log-Poisson structures and the relation with Toda system, and with the post-doc A. Chervov on the Gaudin models and Manin matrices.

References

[1] A. Odesskii, *Elliptic Algebras*, preprint math.QA/0303021 (translated from the Russian: Russ. Math. Surveys 57, no. 6, (2002), 87-122).

[2] A. Odesskii, V. Rubtsov *Integrable systems associated with Elliptic Algebras*, The proceedings of Workshop on Elliptic Integrable Systems, http://www.math.kobe-u.ac.jp/publications/rlm18/18elliptic.html

[3] G. Ortenzi, M. Pedroni, V. Rubtsov *On the Higher Poisson Structures of the CamassaHolm Hierarchy*, Acta Appl. Math., Volume 101, Numbers 1-3 / April, 2008, 243-254.

[4] G. Wilson, *On the quasi-Hamiltonian formalism of the KdV equation*, Phys. Lett. A, Vol 132, n. 8-9