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Studies on a class of one-dimensional integrable shallow water wave models

We mainly study a class of one-dimensional integrable shallow water wave models, typical example is the following Cauchy problem for the Camassa-Holm equation in $\mathbb{R}$

$$\begin{cases} u_t - u_{xxt} + 3uu_x = 2u_x u_{xx} + uu_{xxx}, & t > 0, \ x \in \mathbb{R}, \\ u(x,0) = u_0(x), & x \in \mathbb{R}. \end{cases} \tag{1}$$

This equation, models wave motion in shallow water region with u denoting the height of the water above a flat bottom. Other models in this class are the Degasperis-Procesi equation and the Dullin-Gottwald-Holm equation (DP and DGH for simplicity).

Equation (1) is an integrable system, so there are infinite many conservation laws associated to it. However, unlike the KdV, one of the most significant phenomenon of the Camassa-Holm equation is wave breaking. In this case, the strong (smooth) solution $u(x,t)$ itself remains bounded but its first order derivative $u_x(x,t)$ becomes infinity as (x,t) goes to some point (x_0, t_0). In 1998, H. McKean proved that the solution to (1) breaks down if and only if some portion of the positive part of $y_0(x) = (1 - \partial_x^2)u_0(x)$ lies to the left of some portion of its negative part. But for the DP and DGH equations, to establish the necessary and sufficient condition for wave breaking is still a challenging open problem.

Purpose: One of the purpose of the visit is to cooperate with Professor T. Ratiu at EPFL to make deeper understanding on wave breaking for the DP and DGH equations.

During my visit (15 -29 July 2007), I discussed these problems with Prof. Ratiu and other members visiting the Bernoulli center. We revisited the papers by McKean and some progress was achieved in the understanding of conditions to guarantee the wave breaking in finite time. Based on McKean's papers, we obtained some interesting results and have made some progress on wave breaking for the Degasperis-Procesi equation. That was a very good beginning for next visit.

Results: based on McKean's papers, we fund a very interesting sufficient condition on wave-breaking for the DP equation.

Future collaboration: I hope I can do our future works with Prof. Ratiu at EPFL under the support by ESF. Further work on the DP and DGH equations is our next goal.

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