

Scientific report for exchange grant 1414

ESF Activity “Methods of Integrable systems, Geometry, Applied Mathematics”. Host institution: Imperial College, London

The aim of the visit was to discuss with Dr. John Gibbons and his PhD student Andrea Raimondo some problems concerning the hydrodynamic chains and their reductions.

We focused our attention on the Benney momentum chain, also known the dispersionless limit of the KP hierarchy, and in particular on the Hamiltonian structures of its finite dimensional, hydrodynamic reductions. As well-known such reductions are diagonal integrable systems of hydrodynamic type whose characteristic velocities satisfy a system of partial differential equations, the so called Gibbons-Tsarev system, that involves also the first momentum of the chain.

According to a well-known conjecture due to Ferapontov any diagonal integrable system of hydrodynamic type (called semi-Hamiltonian system) is Hamiltonian w.r.t a suitable, in general non-local, Hamiltonian structure of hydrodynamic type. These structures were introduced by Dubrovin and Novikov, who considered the local case and subsequently generalized by Ferapontov and Mokhov, who considered also the non local case. Ferapontov showed that the non-local tail of such structures can be written explicitly once one knows the quadratic expansion of the non vanishing components of the Riemann curvature tensor in terms of the characteristic velocities of the symmetries of the system. Unfortunately few explicit examples of non local Hamiltonian structures are known in the literature. The main problem is to prove the existence of the above mentioned quadratic expansion.

One of the main result of my collaboration with Andrea Raimondo and prof. John Gibbons is that, in some examples of Benney reduction, this problem can be solved. Moreover, in the examples we considered, the quadratic expansion of the Riemann tensor involves only a finite number of symmetries of the system. We conjecture that it possible to find an infinite number of reductions with this properties and to characterize them explicitly.

We expect to be able to find such reductions and to write a paper on this subject and we plan to continue our collaboration in the future.

During my visit I gave a seminar on the applications of the theory of flat bidifferential ideals to the theory of semi-Hamiltonian systems.