SCIENTIFIC REPORT

June 24 - July 8, 2007

Concerning the visit of Spyridon Kamvissis (Greece) to A.S.Fokas, Cambridge, UK, from June 24 to July 8, 2007.

1. PURPOSE OF THE VISIT

The purpose of the two visits funded by ESF (January-Febrauary and June-July) has been collaboration concerning the asymptotic analysis of the zero dispersion behavior of 2+1-dimensional integrable systems and the related non-local Riemann-Hilbert problem.

2. DESCRIPTION OF THE WORK CARRIED OUT DURING THE VISIT

(a) Riemann-Hilbert problem.

The two main problems appearing in the study of the zero dispersion behavior of 2+1-dimensional integrable systems are (i) the asymptotic analysis of the direct scattering problem, (ii) the asymptotic analysis of the inverse scattering problem, which in some cases is equivalent to a non-local Riemann-Hilbert problem.

The non-local Riemann-Hilbert problem has an interest on its own, due to applications in the theory of biorthogonal polynomilas, the 2-matrix model and non-Hermitian random matrices.

We have considered the following setting: Let Y be a 2x2 matrix, analytic off the real line, with continuous boundary values Y_+, Y_- from above and below the real line, such that

$$Y(z) = [I + O(1/z)]\ diag(z^n, z^{-n})\ at\ \infty,$$

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and

$$Y_+(z) = Y_-(.) \begin{pmatrix} 1 & \Gamma \\ 0 & 1 \end{pmatrix}$$

where $\Gamma(f)$ is the integral operator from $L^2(\mathbb{R})$ to $L^2(\mathbb{R})$, defined by

$\Gamma(f)(x) = \int_{\mathbb{R}} f(y) e^{-(V(x)+W(y)-2\tau xy)} dy$

and V, W are functions with polynomial growth at ∞, while τ is real.

We have been able to construct a g-function transformation that asymptotically (for large n) reduces the above to a scalar non-local Riemann-Hilbert problem on a hyperelliptic Riemann surface, that is explicitly solvable in terms of an integral formula involving the generalized Cauchy kernel associated to the hyperelliptic curve and giving the solution of a scalar singular integral equation.

Our method works for both the case of both fixed and varying (with n) exponential weights, so it will lead to universality results for the associated random matrix problem.

(b) The Dirichlet to Neumann map for NLS.

In recent work, A.Fokas, A.Boutet de Monvel and V.Kotlyarov have been able to reduce the initial boundary value problem for the NLS equation

$$iu_t + u_{xx} + |u|^2 = 0,$$

$$u(x, 0) = u_0(x), u(0, t) = g_0(t),$$

where u_0 is Schwartz and g_0 is periodic, to a Riemann-Hilbert factorization problem in the complex plane. So far, their theory involves use of the extra data $u_x(0, t) = g_1(t)$ which, however, is implicitly defined by the solution of the problem above. A major remaining issue is the study of the so-called Dirichlet-to-Neumann map $g_0 \to g_1$. In particular, one would like to show that if g_0 is periodic (or more precisely "finite gap") then g_1 is finite gap + Schwartz.

Together with A.Fokas, we have set up a plan that will settle this problem once the details are executed.

3. DESCRIPTION OF THE MAIN RESULTS OBTAINED

(i) The non-local Riemann-Hilbert problem appearing in the theory of biorthogonal polynomials and of 2+1 dimensional completely integrable equations is asymptotically reducible to an explicitly solvable one with the aid of the theory of scalar Riemann-Hilbert problems on a hyperelliptic curve.

(ii) The Dirichlet problem for the NLS equation on the first quadrant is solvable via a Riemann-Hilbert factorization problem in the complex plane, when the boundary data $u(0,t)$ are periodic. Indeed, we can show that in such a a case $u_x(0,t)$ is finite gap + Schwartz, hence we can use the formalism developed by Fokas and others.

4. FUTURE COLLABORATION WITH HOST INSTITUTION

Not planned yet.

5. PROJECTED PUBLICATIONS/ARTICLES RESULTING FROM OUR GRANT

S.Kamvissis, Asymptotic Study of a Nonlocal Riemann-Hilbert Problem, preprint.

Other papers will follow.

Spyridon Kamvissis