PROPOSED PROJECT WORK AND AIM OF THE VISIT TO THE UNIVERSITY OF VIENNA

S. Kamvissis

The purpose of the proposed visit is research in collaboration, which will lead to the sequel of two recent papers [1], [2] (also funded by MISGAM) concerning short range perturbations of the (quasi)periodic Toda lattice.

We consider the stability of the periodic Toda lattice under a short range perturbation. More precisely, let the purely periodic Toda lattice (of period N) be given by the equations

$$\begin{aligned} \dot{b}_n^q &= 2(a_n^q)^2 - 2(a_{n-1}^q)^2, \\ \dot{a}_n^q &= a_n^q(b_{n+1}^q - b_n^q), \\ a_{N+n}^q &= a_n^q, \quad b_{N+n}^q = b_n^q. \end{aligned}$$

Consider also the doubly infinite Toda lattice

$$\begin{aligned} \dot{b}_n &= 2(a_n^2 - a_{n-1}^2), \\ \dot{a}_n &= a_n(b_{n+1} - b_n), \\ n &\in \mathbb{Z}, \end{aligned}$$

with initial data such that the first moment of the difference to the periodic lattice is finite

$$\sum_n |n|(|a_n - a_n^q| + |b_n - b_n^q|) < \infty$$

at time $t = 0$. The question is the behavior of a_n, b_n when $t \to \infty$.

In [1] and [2] we have shown that the limiting behavior is described by a particular modulated lattice. Our method of proof is based on an extension of the (nonlinear) stationary phase method for Riemann-Hilbert problems, to the case where the underlying space is a hyperelliptic curve.

Our next goal is to express the next order terms. In particular, we wish to extract the term of order $t^{-1/2}$ that results from the solution of a "local" Riemann-Hilbert problem across a small cross set on a hyperelliptic curve. That problem reduces to the solution of an ODE on the curve.

REFERENCES

[1] S.Kamvissis, G.Teschl, Stability of Periodic Soliton Equations under Short Range Perturbations, Physics Letters A, v.364, n.6, 2007, pp.480-483.

[2] S.Kamvissis, G.Teschl, Stability of the periodic Toda lattice under short range perturbations, arXiv:0705.0346, submitted to Ann.Math.