Application for a short visit MISGAM grant

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Project Description: large N limit of random matrix model in the multi-cut case.
We consider the partition function of a random matrix model,

$$Z_N = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} \prod_{1 \le j < k \le N} (\xi_j - \xi_k)^2 e^{-N \sum_{j=1}^{N} V(\xi_j)} d\xi_1 \dots d\xi_N = \tag{1}$$

where $V(\xi)$ is a polynomial,

$$V(\xi) = \sum_{j=1}^{2d} t_j \xi^j, \quad t_{2d} > 0. \tag{2}$$

In this project we are interested in the asymptotic expansion of the partition function as $N \to \infty$ in the so called multi-gap case, namely when the support of the equilibrium measure $\psi(\xi)d\xi$ which solves the variational problem

$$F_0 = \underset{\{\psi \ge 0, \int \psi d\xi = 1\}}{\mathrm{M}in} \left[- \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \log|\xi - \eta| \psi(\xi)\psi(\eta) d\xi d\eta + \int_{-\infty}^{+\infty} V(\xi)\psi(\xi) d\xi \right], \tag{3}$$

consists of many intervals $J = \cup_{k=0}^{g} (u_{2k+1}, u_{2k+2})$, $u_1 < u_2 < \dots < u_{2g+2}$. In the one-cut regular case

$$-\frac{1}{N^2} \log Z_N \propto F_0 + \frac{1}{N^2} F_1 + \frac{1}{N^4} F_2 + \dots,$$

that is it the logarithmic of the partition function has a regular asymptotic expansion in powers of $1/N^2$ [BIZ], [EM],[BI] and the terms $F_1, F_2, \dots$ can be determined from F_0 [Ey04]. In the multi-gap case, when the support J consists of $g+1$ intervals, the asymptotic behavior of the partition function Z_N in the large N limit was derived in [BDE] using a saddle point argument

$$Z_N \simeq e^{-N^2 F_0} \theta\left(\frac{N\mathbf{\Omega}}{2\pi}; B\right). \tag{4}$$

In the above formula $\theta(\boldsymbol{z}; B) = \sum_{\boldsymbol{n}\in\mathbb{C}^g} e^{\pi i<\boldsymbol{n},B\boldsymbol{n}>+2\pi i<\boldsymbol{z},\boldsymbol{n}>}$ is the Riemann θ-function defined on the Jacobi variety of the hyperellitpic Riemann surface $y^2 = \prod_{k=1}^{2g+2}(\xi - u_k)$, B is the period matrix with respect to a canonical basis of holomorphic differentials and the vectors $\mathbf{\Omega} = (\Omega_1, \Omega_2, \dots, \Omega_g)$ are given by the relation

$$\Omega_j = 2\pi \int_{u_{2j+1}}^{u_{2g+2}} \psi(\xi) d\xi.$$

In this project we are interesting in deriving higher order corrections to the formula (4).

Bibliography

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