# MISGAM application for a short visit grant

Name: C. Klein
Host Institution: SISSA, Trieste Italy
Hosted by T. Grava

**Description of the proposed project work: numerical study of the small dispersion limit of the Korteweg-de Vries equation near singular points.**

In a previous paper [GK] we have presented a quantitative numerical comparison between the solution of the Cauchy problem for the Korteweg-de Vries (KdV) equation

$$u_t + 6uu_x + \epsilon^2 u_{xxx} = 0, \quad u(x,0) = u_0(x), \tag{1}$$

in the small dispersion limit $\epsilon \to 0$, and the asymptotic formula obtained in the works of Lax and Levermore [LL], Venakides [V2] and Deift, Venakides and Zhou [DVZ] which describes the solution of the above Cauchy problem at the leading order as $\epsilon \to 0$. The asymptotic description of [LL], [DVZ] gives in general a good approximation of the KdV solution, but is less satisfactory near the point of gradient catastrophe of the hyperbolic equation, $u_t + 6uu_x = 0$ [D]. This problem has been addressed by Dubrovin. Furthermore after the point of gradient catastrophe an oscillatory region is formed and the description of [LL], [DVZ] is not very satisfactory at the leading and trailing edge of the oscillatory zone. In this project we address the asymptotic description of the KdV equation in the small dispersion limit at the leading edge of the oscillatory zone. The asymptotic description is obtained via a special solution of the Painlevè II equation.

[DVZ] P. Deift, S. Venakides, and X. Zhou, *New result in small dispersion KdV by an extension of the steepest descent method for Riemann-Hilbert problems*, IMRN **6**, (1997), 285-299.

[D] B. Dubrovin, *On Hamiltonian Perturbations of Hyperbolic Systems of Conservation Laws, II: Universality of Critical Behaviour*, Comm. Math. Phys., **267** (2006), 117.

[LL] P. D. Lax and C. D. Levermore, *The small dispersion limit of the Korteweg de Vries equation, I,II,III*, Comm. Pure Appl. Math. **36** (1983), 253-290, 571-593, 809-830.

[V] S. Venakides, *The Korteweg de Vries equations with small dispersion: higher order Lax-Levermore theory*, Comm. Pure Appl. Math. **43** (1990), 335-361.