

# 1 Objectives of the proposal

There has been an increasing interest in the study of the connections random matrix models with different integrable hierarchies. In fact this is one of the lines of research of the MISGAM project. The discovery more than 15 years ago of the role of some Virasoro invariant solutions of the KdV hierarchy as partitions functions in 2D quantum gravity, connected with the Hermitian matrix model and the Kontsevich model where important findings in this respect.

The deep connection between both areas of research has been deepened by the team of Prof. van Moerbeke in different directions, as is shown in the references below. Recently, the appearance, not only of the KP hierarchy but of the more general multi-component generalization has appeared in their studies. For example in relation with mixed multiple orthogonal polynomials and in the study of certain Dyson's Brownian motions.

On the other hand the Toda hierarchy has also played an important role in matrix models. The corresponding dispersionless limit has also shown its relevance in different frameworks as in the theory of conformal maps.

The aim of the proposed collaboration is to study and try to understand how the Takasaki–Takebe formulation of the multi-component Toda hierarchy may extend the above mentioned results regarding the multi-component KP hierarchy. In particular, it might be of interest to understand the role of string equations and Virasoro constraints in this extended framework. Also the large  $N$ -limit related to the corresponding dispersionless theory, which as was shown recently, is connected with the Whitham hierarchy for the multi-component KP hierarchy may be relevant.

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- M. Adler and P. van Moerbeke, *PDEs for the Gaussian ensemble with external source and the Pearcey distribution*, arXiv:math/0509047v1 [math.PR] 2 Sep 2005.
- Mark Adler, Pierre van Moerbeke and Pol Vanhaecke, *Moment matrices and multi-component KP, with applications to random matrix theory*, arXiv:math-ph/0612064v1.
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