Proposed Project Work for Short Visit Grants

—Studies on a class of one-dimensional integrable shallow water wave models

First, we consider the Cauchy problem for the Camassa-Holm equation in $\mathbb{R}$

$$\begin{cases} u_t - u_{xxt} + 3uu_x = 2u_x u_{xx} + uu_{xxx}, & t > 0,\ x \in \mathbb{R}, \\ u(x,0) = u_0(x), & x \in \mathbb{R}. \end{cases} \tag{1}$$

This equation, models wave motion in shallow water region with u denoting the height of the water above a flat bottom.

Equation (1) is a integrable system, so there are infinite many conservation laws associated to it. However, unlike the KdV, one of the most significant phenomenon of the Camassa-Holm equation is wave breaking. In this case, the strong (smooth) solution $u(x,t)$ itself remains bounded but its first order derivative $u_x(x,t)$ becomes infinity as (x,t) goes to some point (x_0, t_0). In 1998, H. McKean [1] (see [2] for a simple proof) proved that the solution to (1) breaks down if and only if some portion of the positive part of $y_0(x) = (1 - \partial_x^2)u_0(x)$ lies to the left of some portion of its negative part.

One of the goals of the visit is to cooperate with Professor T. Ratiu at EPFL to make deeper understanding on the evolution of the corresponding solution with McKean's initial condition. We will try to give an alternative proof to get more information of the solution as it near the blow-up time. It should be an explicit analysis on how the solution breaks down.

We will also consider other integrable system which are close to the Camassa-Holm equation, such as the Degasperis-Procesi equation and the Dullin-Gottwald-Holm equation. Collaborate with Professor T. Ratiu to do some blow-up analysis for the corresponding solutions.

References

[1] McKean, H. P., Breakdown of a shallow water equation, Asian J. Math., 2 (1998), No.4, 867–874.

[2] McKean, H. P., Breakdown of the Camassa-Holm equation. Comm. Pure Appl. Math. 57 (2004), no. 3, 416–418.