Projetc work and aim of the visit

As it is well known integrable systems of hydrodynamic type naturally arise in several areas of physics such gas dynamics, shallow water theory, nonlinear elasticity, etc.

In the case of diagonalizable systems and in diagonalizing coordinates (called Riemann invariants) the condition of integrability (due to Tsarev) is nothing but the compatibility condition of the systems that provide the symmetries and the densities of conservation laws.

In a generic coordinate system, to check the integrability of a given system of hydrodynamic type one has to check the vanishing of two tensors associated to the system: the vanishing of the first one, called Haantjes tensor, is equivalent to the existence of the Riemann invariants (this is a classical result due to Haantjes), the vanishing of the second one (introduced by Pavlov, Svinolupov and Sharipov) is equivalent to the integrability condition.

A natural generalization of systems of hydrodynamic type are the hydrodynamic chains. In this case the number of equations and variables is infinite. A possible criterion of integrability for hydrodynamic chain, proposed by Ferapontov and Marshall, is based on the observation that, under some additional reasonable assumptions, the components of the two tensors mentioned above are sum of a finite number of terms and therefore, at least in principle, it is possible to compute them explicitly. Probably the most known example of hydrodynamic chain is the Benney momentum chain, introduced by Benney in the study of long nonlinear waves.

In many important cases integrable systems of hydrodynamic type can be obtained as finite component reductions of hydrodynamic chains (for istance the reductions of the Benney chain have been extensively studied by Gibbons and Tsarev).

The aim of my research programme is to study the applications of certain mathematical structures, appearing naturally in the theory of separation of variables, to integrable systems of hydrodynamic type. The first example of such applications has been found by Ferapontov and Fordy and it is related to the Stäckel theory of separation of variables. Recently, in collaboration with Franco Magri, I started to study a special class of integrable systems of hydrodynamic type (containing the main example of Ferapontov and Fordy) that have been obtained previously by Pavlov as reductions of a hydrodynamic chain. The starting point of the joint work with Magri was the observation that the integrability condition, for Pavlov reductions has a simple interpretation in terms of bidifferential ideals. Bidifferential ideals are ideals of differential forms which are closed with respect two differentials: the

first one is the usual Cartan differential, the second one can be constructed starting from a tensor of type $(1,1)$ with vanishing Nijenhuis torsion. Despite the abstract language the applications of the theory of bidifferential ideals to integrable systems of hydrodynamic type are rather concrete. Indeed it naturally provides a recursive procedure to compute the symmetries and the densities of conservation laws.

As I showed in a recent paper the class of integrable systems of hydrodynamic type related to the theory of flat bidifferential ideals contains also other important examples such as the genus one Whitham equations for KdV. Moreover the metrics defining the Hamiltonian structures of these systems satisfy a system of Egoroff-Darboux type which appears also in the theory of Frobenius manifolds. This fact suggests a possible relation (probably more involved that in the cases mentioned above) between the theory of bidifferential ideals and the systems of hydrodynamic type related to the theory of Frobenius manifolds.

Summarizing I plan to deepen the study of the applications of bidifferential ideals to integrable systems of hydrodynamic type. I think that this theory should have some applications also in the theory of integrable hydrodynamic chains and I plan to make some attempts in this direction.

The aim of my visit is to discuss some points of the above project work with prof. Gibbons, who is an expert in the field of integrable systems of hydrodynamic type, hydrodynamic chains and their reductions.