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Research project

During my stay at SISSA I am planning to study jointly with Prof. Ugo Bruzzo some topics in the geometry of Higgs bundles. This will be based on previous results that we have already obtained in collaboration.[1]

In a paper by U. Bruzzo and D. Hernández Ruipérez[2] a numerical characterization was given of those Higgs bundles on complex projective manifolds that satisfy the properties of being semistable and having vanishing discriminant (where the discriminant of a rank r bundle is the characteristic class $\Delta(E) = 2rc_2(E) - (r-1)c_1(E)^2$). Building on this result we have provided a notion of numerical effectiveness for Higgs bundles on complex projective manifolds, proving several properties of such bundles. An interesting consequences of these results is the fact that these Higgs bundles are exactly those that are semistable after restriction to any curve in the base manifold (one should compare here with the Mehta-Ramanathan theorem about restrictions of semistable (Higgs) bundles). Moreover these Higgs bundles turn out to be semistable for *every* choice of a polarization in the base manifold.

These constructions have then been extended to the case of compact Kähler manifolds. The key idea here has been to provide notions of numerical effectiveness in terms of fibre metrics, and to import techniques from usual bundle theory by replacing the Chern connection with the Hitchin-Simpson connection, which is defined in terms of the Higgs field.

This background opens the way to further investigations.

1. There is a relationship between the semistability of a vector bundle and the existence of an approximate Hermitian-Yang-Mills structure on it. This is known after the work of Kobayashi and has been proved for vector bundles on projective manifolds. We believe that our techniques may used to extend this result to the case of Higgs bundles, and more notably, to the case of nonprojective Kähler base manifolds.

2. Semistable (Higgs) bundles with vanishing discriminant have the properties of being semistable after restriction to any closed subvariety of the base variety. Thus such bundles are very special and in many cases (i.e., for some special choices of the base variety) may be explicitly characterized. We plan to give such a characterization, and to apply it to the classification of those projective varieties whose tangent bundle is of this type.

3. Numerical chacterizations of semistability, and related notions of numerical effectiveness, exist in the literature for vector bundles, principal bundles and Higgs bundles. We plan to cover also the case of principal Higgs bundles (i.e., principal bundles P with a choice of an element ϕ in $\Omega^1(\mathrm{ad}P)$ such that $\phi \wedge \phi = 0$).

4. We want to give a characterization of semistability for Higgs bundles E in terms of the numerical effectiveness of a class related to the universal object on the complete flag variety of the bundle E. This will open the way to the introduction of some tautological line bundles on the moduli space of semistable Higgs bundles. We shall investigate applications of this construction to the Hitchin completely integrable system.

[1] Bruzzo, U. and Graña Otero, B. Numerically flat Higgs vector bundles. Submitted to Communications in Contemporary Mathematics. SISSA Preprint June 2005; Bruzzo, U and Graña Otero, B. Metrics on semistable and numerically effective Higgs bundles. J. reine ang. Math. Accepted September 2006.

[2] Semistability vs. nefness for (Higgs) vector bundles, Differential Geom. Appl. **24** (2006), 403–416