PROPOSED PROJECT WORK AND AIM OF THE SECOND VISIT TO CAMBRIDGE

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The purpose of this SECOND visit (June 24 to July 7, 2007) to Cambridge, and in particular the Isaac Newton Institute, concerns:

1. Work with A.Fokas on the semiclassical analysis of 2+1 dimensional integrable equations. I note here that I have just submitted a first application for a short visit from January 21 to February 4, 2007. I believe that a second short visit will be necessary after a few months.

2. Participation in a workshop at the Isaac Newton Institute on Highly Oscillatory Problems. In particular, I would like to present new results on the nonlinear steepest descent method and also benefit from discussions with experts.

Concerning the work with Fokas, and as already stated in my first application, the proposed model is the Davey-Stewartson equation in the semiclassical limit, i.e.

$$
\begin{aligned}
i\epsilon\tilde{q}_\tau + \epsilon^2\tilde{q}_{rr} + \epsilon^2\tilde{q}_{ss} + (\tilde{U}_1 + \tilde{U}_2)\tilde{q} = 0,\\
\tilde{U}_1 = \frac{1}{2}\int_{-\infty}^{r} dr'(|\tilde{q}|^2)_s + \tilde{u}_1(s,\tau),\\
\tilde{U}_2 = \frac{1}{2}\int_{-\infty}^{s} ds'(|\tilde{q}|^2)_r + \tilde{u}_2(r,\tau).
\end{aligned}
\tag{1}
$$

under initial data $\tilde{q}(r,s,\tau=0) = q_0(r,s)$. The parameter ϵ is meant to be small, compared to r, s, τ.

Even though the semiclassical analysis of 1+1 dimensional integrable equations has been very succesful (see e.g.[LL], [JLM], [KMM]) the outstanding 2+1 dimensional problem is still unsolved. The main difficulty is the semiclassical study of the inverse scattering problem which is now a non-local Riemann-Hilbert problem.

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Motivated by the ideas of [MK] we propose to tackle the problem above by first focusing on the very special potential $-N(N+1)sech^2(\xi)$ for the "boundary" functions u_1, u_2.

More precisely, introducing the transformation

$$\begin{aligned} \xi = \frac{r}{\epsilon}, \eta = \frac{s}{\epsilon}, t = \frac{\tau}{\epsilon}, \\ q(\xi,\eta,t) = \tilde{q}(r,s,\tau), \\ U_j(\xi,\eta,t) = \tilde{U}_j(r,s,\tau), \\ u_1(\eta,t) = \tilde{u}_1(s,\tau), u_2(\xi,t) = \tilde{u}_2(r,\tau), \end{aligned} \tag{2}$$

we have the standard DS I equation

$$\begin{aligned} & iq_t + q_{\xi\xi} + q_{\eta\eta} + (U_1+U_2)q = 0, \\ & U_1 = \frac{1}{2}\int_{-\infty}^{\xi} d\xi'(|q|^2)_\eta + u_1(\eta,t), \\ & U_2 = \frac{1}{2}\int_{-\infty}^{\eta} d\eta'(|q|^2)_\xi + u_2(\xi,t). \end{aligned} \tag{3}$$

under initial data $q(\xi,\eta,t=0) = q_0(\epsilon\xi,\epsilon\eta)$. If $r,s,\tau = O(1)$, then the interesting range for ξ,η,t is $O(1/\epsilon)$, as $\epsilon \to 0$.

We consider the case where u_j are independent of time, for the moment.

In particular, let $q_0(a,b) = sech(a)sech(b)$, and

$$\begin{aligned} \tilde{u}_1(s) = -(1+\epsilon)sech^2(s), \\ \tilde{u}_2(r) = -(1+\epsilon)sech^2(r), \end{aligned} \tag{4}$$

so that

$$\begin{aligned} q(\xi,\eta,t=0) = sech(\epsilon\xi)sech(\epsilon\eta), \\ u_1(\eta) = -(1+\epsilon)sech^2(\epsilon\eta), \\ u_2(\xi) = -(1+\epsilon)sech^2(\epsilon\xi). \end{aligned} \tag{5}$$

We need to solve the associated linear system ((2.1) of [FS]) or the equivalent Volterra system ((2.3) of [FS]). A special role is played by the eigenfunctions of the Schrödinger operator with potentials u_1, u_2.

For the rescaled potential $-N(N+1)sech^2(\xi)$, the eigenfunctions can be explicitly expressed in terms of the hypergeometric function. This, together with the simple evolution in time (as studied in [FS]) enables us to express the solution q (asymptotically) in terms of the solution of a linear system of equations.

We expect that the analysis of the explicit (but complicated in the limit) formulae for q can be performed as $\epsilon \to 0$, thus providing a breakthrough for the problem. This is the plan of our collaboration.

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