

Scientific report for ESF grant on 'Methods of Integrable Systems, Geometry, Applied Mathematics'

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Host institution: Sissa, Trieste, Italy

Hosted by: Prof. Tamara Grava

Purpose of the visit

The purpose of the visit was to exchange some ideas with Prof. Tamara Grava and Prof. Boris Dubrovin concerning critical random matrix ensembles, the small dispersion limit of the Korteweg-de Vries equation, the fourth order analogue of the Painlevé I equation and Riemann-Hilbert problems.

Work carried out during the visit and description of the main results obtained

During the first weeks of my visit, I gave a mini-course consisting of three lectures on the topic 'Riemann-Hilbert problems and random matrices'.

Furthermore we discussed several possible research problems which we could try to solve. We decided to study the behavior of a solution of the KdV equation near the point of gradient catastrophe in the small dispersion limit. Our aim was to give small dispersion asymptotics for a solution in a rigorous way, by applying the Deift/Zhou steepest descent method on the Riemann-Hilbert problem for the KdV equation. We already solved a significant part of this problem, although the work is not finished yet. We expect to be able to write a paper on this subject in the near future.

Another problem we studied concerns the fourth order analogue of the Painlevé I equation. This equation has a unique real pole-free solution which has interesting applications both in the theory of random matrices and integrable systems. This solution is expected to show different asymptotic regimes, such as algebraic asymptotics, elliptic asymptotics, Painlevé II asymptotics and a last type of asymptotics for which it is not known yet what to expect. We discussed the possibility of finding these asymptotics rigorously using the Riemann-Hilbert approach. This did not result yet in any concrete results, but at least we have been able to exchange several interesting ideas.

Besides the research problems we studied, I have also been able to attend several interesting lectures and to learn more about various areas in mathematics and mathematical physics.