Scientific report for short visit grants MISGAM-1171

— Blow-up analysis for the Camassa-Holm equation

We consider the Cauchy problem for the Camassa-Holm equation in $\mathbb{R}$

$$\begin{cases} u_t - u_{xxt} + 3uu_x = 2u_x u_{xx} + uu_{xxx}, & t > 0,\ x \in \mathbb{R}, \\ u(x,0) = u_0(x), & x \in \mathbb{R}. \end{cases} \tag{1}$$

This equation, models wave motion in shallow water region with u denoting the height of the water above a flat bottom.

Equation (1) is an integrable system, so there are infinite many conservation laws associated to it. However, unlike the KdV, one of the most significant phenomenon of the Camassa-Holm equation is wave breaking. In this case, the strong (smooth) solution $u(x,t)$ itself remains bounded but its first order derivative $u_x(x,t)$ becomes infinity as (x,t) goes to some point (x_0,t_0). In 1998, H. McKean proved that the solution to (1) breaks down if and only if some portion of the positive part of $y_0(x) = (1-\partial_x^2)u_0(x)$ lies to the left of some portion of its negative part.

Purpose: One of the purpose of the visit is to cooperate with Professor T. Ratiu at EPFL to make deeper understanding on the evolution of the corresponding solution with McKean's initial condition. Another one is to give a description of the profile of the corresponding solution at the blow-up time. Moreover, we also consider the Degasperis-Procesi equation.

During my visit (23 October-5 November 2006), I discussed these problems with Prof. Ratiu and other members visiting the Bernoulli center. We revisited the papers by McKean and some progress was achieved in the understanding of conditions to guarantee the wave breaking in finite time. Based on McKean's papers, we obtained some interesting results and can give an alternative explanation of McKean's papers. We also have made some progress on wave breaking for the Degasperis-Procesi equation. That was a very good beginning for next visit.

Results:

1. Based on McKean's papers, we prepared to write an alternative proof for McKean's necessary and sufficient condition on wave-breaking.

2. We wrote a paper "A note on wave breaking for the Camassa-Holm equation", in which we proved that for any initial datum u_0 with compact support, the corresponding solution $u(x,t)$ to (1) blows up in finite time.

Future collaboration: I hope I can do our future works with Prof. Ratiu at EPFL under the support by ESF. The Degasperis-Procesi equation is our next goal.

Article: A note on wave breaking for the Camassa-Holm equation, preprint. Support from ESF was acknowledged.

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