Final report on the short visit grant
Reference Number: 1390

Purpose of the visit: The purpose of my visit in LPT, Université de Paris-Sud was the collaboration with Prof. Bernard Jancovici on a generalization of the quantum Wigner-Kirkwood expansion to classically integrable statistical Coulomb models with a boundary.

Description of the work carried out during the visit: The topic of my primary interest is the exact solution of the statistical mechanics for classical two-dimensional Coulomb fluids of charged particles interacting in pairs via logarithmic forces, in terms of their equivalence with integrable (1+1)-dimensional field theories like the sine-Gordon or Bullough-Dodd models. Exact results involve infinite in space (bulk) Coulomb fluids and special cases of semi-infinite Coulomb fluids in contact with a dielectric hard wall impenetrable to particles, which are mappable onto integrable boundary sine-Gordon theories.

The exact treatment of the statistical mechanics for the classical version of a fluid is a first step in constructing the exact solution of the statistical mechanics for the quantum version of this fluid. In the case of bulk quantum fluids of particles interacting via pairwise interactions, the Wigner-Kirkwood formalism provides a semiclassical expansion of the Boltzmann density in configuration space in even powers of the Plank's constant $\hbar$ (or, equivalently, the thermal de Broglie wavelength λ). This result permits one to generate an analogous λ-expansion for the bulk free energy and many-body densities.

To my knowledge, there does not exist an extension of the bulk Wigner-Kirkwood expansion to boundary statistical quantum models. The aim of the work carried out in collaboration with B. Jancovici was a generalization of the Wigner-Kirkwood technique to semi-infinite quantum fluids, constrained by a hard wall.

Description of the main results obtained: Using a Laplace transform method, we have worked out a nontrivial generalization of the bulk Wigner-Kirkwood expansion to semi-infinite quantum fluids by expressing the Boltzmann density in the basis of the stationary wave functions. In contrast to the bulk case, the resulting Boltzmann density involves also position-dependent terms which are non-analytic in λ. The analyticity in λ is restored by integrating the Boltzmann density over configuration space; however, in contrast to the bulk free energy, the semiclassical expansion of the surface part of the free energy (surface tension) contains also odd powers of λ. The formalism permits to derive the leading quantum corrections for one-body and two-body densities close to the system boundary. As a model system for explicit calculations, we used the one-component Coulomb plasma.

Future collaboration with host institution: The standard Wigner-Kirkwood expansion for infinite systems neglects fermion or boson exchange effects between quantum particles. For Coulomb bulk systems, the exchange fermion effects induce corrections which are exponentially small in λ, and therefore can be neglected. We have to check this phenomenon also for boundary Coulomb fluids. This is why I plan a short-term stay at LPT also the next year.

Projected publications/articles resulting from the grant: We are now in the process of writting a publication about the obtained results. The publication will be probably submitted to the "Journal of Statistical Mechanics" (JSTAT) published jointly by IOP and SISSA.

Other comments: I have no comments.