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3)PROJECT: INTEGRABLE DYNAMICAL SYSTEMS, THEIR NONAUTONOMOUS PERTURBATIONS AND RELATED ERGODIC PROPERTIES

RESEARCH TOPICS:

1. THE DIFFERENTIAL-GEOMETRIC ASPECTS OF QUASI-ISOSPECTRAL DYNAMICAL SYSTEMS AND APPLICATIONS

Some aspects of the description of Lagrangian and Hamiltonian formalisms naturally arising from the invariance structure of given nonlinear dynamical systems on the infinite–dimensional functional manifold is presented. The basic ideas used to formulate the canonical symplectic structure are borrowed from the Cartan's theory of differential systems on associated jet–manifolds. The symmetry structure reduced on the invariant submanifolds of critical points of some nonlocal Euler–Lagrange functional is described thoroughly for both differential and differential discrete dynamical systems. The Hamiltonian representation for a hierarchy of Lax type equations on a dual space to the Lie algebra of integral-differential operators with matrix coefficients, extended by evolutions for eigenfunctions and adjoint eigenfunctions of the corresponding spectral problems, is obtained via some special Backlund transformation. The connection of this hierarchy with integrable by Lax spatially two-dimensional systems is studied. One of the fundamental problems in modern theory of infinite-dimensional dynamical systems is that of an invariant reduction them upon some invariant submanifolds with enough rich mathematical structures to treat their properties analytically. The first approaches to these problems were suggested still at the late times of the preceding century, in the classical oeuvres by S.Lie, J.Liouville, J.Lagrange, V.R.Hamilton, J.Poisson and E.Cartan. They introduced at first the important concepts of symmetry, conservation law, symplectic, Poisson and Hamiltonian structures as well invariant reduction procedure, which appeared to be extremely useful for proceeding studies. These notions were widely generalized further by Souriau [21], Marsden and Weinstein [34, 20], Lax [3], Bogoyavlensky and Novikov [7], as well by many other researchers [8, 10, 11, 12, 13]. It seems worthwhile to mention here also the recent enough studies in [22, 23, 24, 39, 26, 27, 28, 29], where the special reduction methods were proposed for the integrable nonlinear dynamical systems on both functional and operator manifolds. In the present paper we describe in detail the reduction procedure for infinite dimensional dynamical systems upon the invariant set of critical points of some global invariant functional. The method uses the Cartan's differential-geometric treating of differential ideals in Grassmann algebra over the associated jet–manifold. As one of main results, we show also that both the reduced dynamical systems and their symmetries, generate the Hamiltonian ows on the invariant critical submanifolds of local and nonlocal functionals with respect to the canonical symplectic structure upon it.

These results are generalized for the case of differential-difference dynamical systems being given on discrete infinite-dimensional manifolds. The direct procedure to construct the invariant Lagrangian functionals for a given apriori Lax-type integrable dynamical system is presented for both the differential and the differential-difference cases of the manifold M. Some remarks on the Lagrangian and Hamiltonian formalisms, concerned to infinite-dimensional dynamical systems with symmetries are given. The Hamiltonian representation for a hierarchy of Lax type equations on a dual space to the Lie algebra of integral-differential operators with matrix coefficients, extended by evolutions for eigenfunctions and adjoint eigenfunctions of the corresponding spectral problems, is obtained via some special Backlund transformation. The connection of this hierarchy with integrable by Lax spatially two-dimensional systems is studied.

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2. THE ERGODIC MEASURES RELATED WITH NONAUTONOMOUS HAMILTONIAN SYSTEMS AND THEIR HOMOLOGY STRUCTURE

The past years have given rise to several exciting developments in the field of symplectic geometry and dynamical systems [3-12], which introduced new mathematical tools and concepts suitable for solving numerous problems wich were earlier intractable. When studying periodic solutions to non-autonomous Hamiltonian systems, Salamon & Zehnderdeveloped a proper Morse theory for infinite dimensional loop manifolds based on previous results on symplectic geometry of Lagrangian submanifolds of Floer. Investigating at the same time ergodic measures related with Lagrangian dynamical systems on tangent spaces to configuration manifolds, Mather devised a new approach to studying the correspondingly related invariant probabilistic measures based on a so called β-function. The latter made it possible to describe effectively the so called homology of these invariant probabilistic measures minimizing the corresponding Lagrangian action functional.

As one can easily see, Mather's approach does not allow any direct application to the problem of describing the ergodic measures naturally related to a given periodic non-autonomous Hamiltonian system on a closed symplectic space. Thereby, to overcome constraints to this task in the present work we suggest some new way to imbedding the non-autonomous Hamiltonian case into the Mather β-function theory picture, making use of the mentioned above Salamon & Zehneder and Floer loop space homology structures. Further, the Gromov elliptic techniques in symplectic geometry make it possible to construct the invariant submanifolds of our Hamiltonian system, naturally related to the corresponding compact Lagrangian submanifolds, and a β-function analog on them.