PROJECT WORK of MATTEO PETRERA

Integrable discretizations of extended Lagrange tops

A project on the integrable time discretization of the recently discovered hierarchy of extended Lagrange tops has been initiated. The work is now in progress with the collaboration of Y. Suris and O. Ragnisco.

Suitable algebraic procedures performed on N–site Gaudin models associated with any simple Lie algebra $\mathfrak{g}$ and with any dependence on the spectral parameter, allow us to construct a hierarchy of integrable models sharing the same (linear) r–matrix structure [1]. In the rational case, with $\mathfrak{g} = \mathfrak{su}(2)$, we have obtained a new family of *extended Lagrange tops* [2]. They provide an example of rigid body dynamics described by a heavy symmetric top with $N-2$ interacting heavy satellites. If $N = 2$ we recover the well known Lagrange top. Furthermore, a direct sum procedure allows us to build long–range chains of interacting bodies, with rational, trigonometric and elliptic r–matrices.

The approach to integrable discretization proposed by Moser and Veselov [3] and developed by other authors (see [4]) is a powerful tool to construct integrable maps studying Lagrangian and Hamiltonian properties of integrable systems when the basic manifold carries the additional structure of a Lie group.

Our starting point is the result contained in [5]. Here the authors propose an exact integrable discretization for the Lagrange top ($N = 2$ in our framework), providing a discrete Lax representation of the map and a discrete Lagrangian formulation.

In the same spirit we have obtained a discrete integrable map for our $\mathfrak{su}(2)$ hierarchy. We have proposed a proof for its Poisson property, but the integrability of the map remains elusive. Nevertheless an evidence for the integrability property has been obtained with the help of Maple 6 up to the 7th extension. Hence we are now working in order to get a Lax representation for our map.

A discrete Poisson map for the so called rational Lagrange chain [6] has been also proposed. Its integrability has to be investigated and the work is now in progress.

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[1] F. Musso, M. Petrera, O. Ragnisco, Jour. Nonlin. Math. Phys. **12**, Suppl. 1 (2005).
[2] F. Musso, M. Petrera, O. Ragnisco, G. Satta, Reg. Chaot. Dyn. **10**, Issue 4 (2005).
[3] J. Moser, A.P. Veselov, Comm. Math. Phys. **139** (1991).
A.P. Veselov, Uspekhi Mat. Nauk. **46** (1991).
A.P. Veselov, Comm. Math. Phys.**145** (1992).
[4] Y. Suris, The problem of integrable discretization: Hamiltonian approach, Birkhäuser Verlag, Basel, 2003.
[5] A.I. Bobenko, Y. Suris, Amer. Math. Soc. Transl. **201** (2000).
[6] F. Musso, M. Petrera, O. Ragnisco, G. Satta, Jour. Nonlin. Math. Phys. **11**, Suppl. 2 (2005).