RIEMANN-HILBERT PROBLEM FOR Z_N CURVES AND APPLICATIONS

VISIT TO SISSA OF V.ENOLSKI WITHIN MISGAM PROGRAM

1. Description of the project and aim of the visit

We are planning to continue investigations started in the article of the applicant and T.Grava, *Riemann-Hilbert problem of the singular Z_N-curve* IMRN, 32, 1619-1683(2004) where Riemann-Hilbert problem was solves for singular Z_N curves and monodromy given by $N \times N$ quasi-permutation matrices. We shall extend this analysis to more complicated case of general Z_N-curves. Our method requires solution of associated algebro-geometeric problems like explicit description of homologies, non-special divisors supported by branch points, $1/N$ and $1/2N$ theta-characteristics. We shall construct Riemann period matrix and Baker-function in terms of theta-functions. We shall also derive theta-constant relations of Thomae type, involving of multivariable hypergeometeric functions and relations between them. The Thomae type formulae for non-hyperelliptic curves were recently considered by the applicant and T.Grava in *Thomae type formulae for singulae Z_N-curve* arXiv: math-ph/0602017 these results will be used in present project. Our investigation will be supported by computer algebra means which were developed when we were working on the aforementioned publications.

As a particular case the cyclic triple covering of the complex plane branched in six points will be considered. This case has already rich geometric structure and admits introduction of further symmetries. On this stage we are planning to develop Weierstrass-Poincaré reduction to describe explicitly moduli space and covers of associated curves. Analysis of the aforementioned curve interrelated with recent results of the applicant and H.Braden, *Remarks on the complex geometry of 3-monopole* arXiV: math-ph/0601040 where 3-monopole solutions to the Bogomolny equations were described from the unified theta-functional view point. Result of this work, methods and computer algebra sofwares shall be implemented in the present project. Clarifying of interrelations of both problems would be also of the interest for the prsent program.

As an application we shall consider isomonodromic deformation and present associated Schlesinger equation and its theta-functional solutions. The trigonal curve of genus four, which is the first nontrivial curve will considered in most details. In this case standard and multivariable hypergeometeric functions are planned to be used.

Another fundamental application be derivation of asymptotic of biorthogonal polynomials within algebro-geometric approach. That's the problem of modern interest which demands consideration of non-hyperelliptic curves and their theta-functions.

The problems described in the program is not supposed to be completed within 6-weeks visit: we descrive a program of much longer collaboration. The aim of the visit is to start work in the pointed direction and obtain first results. The applicant is also planning to discuss his recent results in theory of magnetic monopole and give a talk on this subject.