Integrable dispersionless systems and string equations

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I am young researcher who is involved in research and development on integrable nonlinear Hamiltonian infinite-dimensional systems. This issue belongs to the problem considered by European Science Foundation (ESF) Scientific Programme: Methods of Integrable Systems, Geometry, Applied Mathematics (MISGAM).

Nonlinear evolution equations describe many physical phenomenons. However, most of them manifest chaotic behaviour. Therefore studying of integrable nonlinear systems is of particular importance for understanding at least of some aspects of nonlinear equations.

The theory of integrable nonlinear dispersionless field systems (the so-called hydrodynamic systems) belongs to the most recent ones and has been systematically developing from the 80s of the last century. As integrable systems we understand those which have an infinite hierarchy of symmetries. In the beginning the theory was mainly developed by Russian school, represented by such researchers as: Novikov, Dubrovin, Tsarev, Mokhov, Ferapontov, Pavlov and others, and at present it is developed at academic centers all over the world. The group from Madrid, I am going to visit, is one of the leading in the field. The intensive research of integrable dispersionless systems started since Tsarev developed the technique of linearization of an appropriate dispersionless systems, written down in the so-called Riemann invariants, through the so-called hodographic transformation, and then showed the way of finding solutions, with the use of quadratures. The multi-Hamiltonian theory of such systems in terms of the Riemann geometry has been developed during last decade by the Russian school.

During last few years significant input in development of theory of integrable dispersionless systems was done by the Madrid group: L.M. Alonso, M. Manas and by their coauthors [1-2]. Among others, they apply and develop the formalism of dressing operators, reductions within hodograph solutions of the dispersionless systems based on so-called *S*-function with the aim of finding solution to the appropriate integrable dispersionless equations. It is important here to notice, that most recent articles [1] written by M. Manas apply above methods to the hierarchies introduced by myself and M. Blaszak in [3].

In the theory of evolutionary systems one of the most important issues is a systematic method for construction of integrable systems. It is well known that a very powerful tool, called the classical *R*-matrix formalism, proved to be very fruitful in systematic construction of the field and lattice soliton systems as well as dispersionless systems. The crucial point of the formalism is the observation that integrable dynamic systems can be obtained from the Lax equations on appropriate Lie algebras. The biggest advantage of this formalism, besides systematic construction of the integrable systems, is the possibility of the construction of bi-Hamiltonian structures, infinite hierarchies of symmetries and conserved quantities. Recently the theory of *R*-matrices for integrable hydrodynamic systems, based on the commutative algebra of the Lax functions has been developed by myself and M. Blaszak [3].

The theory of integrable field (lattice) systems with dispersion, the so-called soliton systems, has been systematically developing from the beginning of the 70s of the last century. As well known, a quasi-classical limit of field and lattice soliton systems gives a related integrable dispersionless systems. One can inverse this procedure and construct field and lattice soliton systems from some classes of integrable dispersionless systems through a Weyl-Moyal like deformation quantization procedure. Actually, it can be done on the level of their Lax representations (see my article with M. Blaszak [3]).

During my visit to the Department of Theoretical Physics of Complutense University in Madrid I would like to concentrate on the following three aspects:

1. As I am a Ph.D. student, first of all I am interested in study of some advanced methods in the theory of integrable nonlinear dynamic systems and take advantage from the experience of the Madrid group. Next, on the basis of our knowledge I would like to investigate the following problems:
2. The application of *R*-matrix approach to the so-called universal hierarchy considered recently by Alonso and Shabat [4] in order to develop the problem using different formalism and search for a possible Hamiltonian structures of related systems.
3. The application of hodograph formalism, *S*-functions etc. (develop by the Madrid group) with the aim of solving integrable dispersionless systems with the rational Lax representations. Such systems are recently considered by myself and M. Blaszak. As well, the construction of dispersionless Lax hierarchies constraint by string equations.
4. To find some relations between the so called universal hierarchy [4] and finite dimensional, separable Stackel systems.

I believe that the visit to Madrid will improve my scientific experiences and knowledge, as the one of main scopes of the MISGAM program is to create a fruitful training ground for young researchers. I also believe, that the results of our common research will contribute some new results to the theory of integrable nonlinear evolutionary systems.

1. Manas M, Alonso M L and Medina E "Dressing methods for geometric nets. I. Conjugate nets. II. Orthogonal and Egorov nets" *J. Phys. A: Math. Gen.* **33** (2000) 2871-94, 7181-206; "Reductions and hodograph solutions of the dispersionless KP hierarchy" *J. Phys. A: Math. Gen.* **35** (2002) 401-17
2. Manas M "On the r-th dispersionless Toda hierarchy: factorisation problem, additional symmetries and some solutions" *J. Phys A: Math. Gen.* **37** (2004) 9195-224; "*S*-functions, reductions and hodograph solutions of the r-th dispersionless modified KP and Dym hierrachies" *J. Phys A: Math. Gen.* **37** (2004) 11191-221
3. Blaszak M and Szablikowski B M "Classical *R*-matrix theory of dispersionless systems: I. (1+1)-dimension theory. II. (2+1)- dimension theory" *J. Phys A: Math. Gen.* **35** (2002) 10325-44, 10345-64; "From dispersionless to soliton systems via Weyl-Moyal-like deformations" *J. Phys A: Math. Gen.* **36** (2003) 12181-203
4. Alonso M L and Shabat A B "Hydrodynamic reductions and solutions of the universal hierarchy" (Russian) *Teoret. Mat. Fiz.* **140** (2004) 216-29; "Energy-dependent potentials revisited: a universal hierarchy of hydrodynamic type" *Phys. Lett. A* **300** (2002) 58-64

The Aim of the visit

The aim of the visit to the Department of Theoretical Physics of Complutense University in Madrid is to take advantage of the experience of the Madrid group and improve my scientific experiences and knowledge. I also expect to be able to begin a mutually profitable investigation of some specific problems from the theory of integrable nonlinear systems.

List of publications

1. Błażej M. Szablikowski and Maciej Błaszak, "On deformations of standard R-matrices for integrable infinite-dimensional systems", *J. Math. Phys.* **46** (2005) 42702

2. Błażej M. Szablikowski and Maciej Błaszak, "Soliton Systems as star-Deformed Dispersionless Equations", *Proceedings of Institute of Mathematics of NAS of Ukraine* **50** (2004) Part 1, 478-485

3. Maciej Błaszak and Błażej M. Szablikowski, "From dispersionless to soliton systems via Weyl-Moyal-like deformations", *J. Phys. A: Math. Gen.* **36** (2003) 12181-12203

4. Maciej Błaszak and Błażej M. Szablikowski, "Classical R-matrix theory of dispersionless systems: II. (2+1)-dimension theory", *J. Phys. A: Math. Gen.* **35** (2002) 10345

5. Maciej Błaszak and Błażej M. Szablikowski, "Classical R-matrix theory of dispersionless systems: I. (1+1)-dimension theory", *J. Phys. A: Math. Gen.* **35** (2002) 10325