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Poisson cohomology and Deformations

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Application for an ESF Exchange Grant

1 Aim of the visit and research project

I plan to spend three weeks from the 26th of April to the 14th of May 2005 in the University of Louvain-La-Neuve, Belgium.

I am working on a PhD thesis, on Poisson cohomology and Deformations, under the supervision of Professor Pol Vanhaecke. The Poisson algebras first appear in classical mechanics with D. Poisson who defined a bracket of functions in $\mathbf{R}^{2r}$, that permitted to express the Hamiltonian equations as differential equations where the positions and the impulsions play symmetric roles. For each Poisson algebra, one define a cohomology, called Poisson cohomology, that have been introduced by A. Lichnérowicz. In fact, Poisson cohomology gives very interesting information about the Poisson structure and about deformations. The problem is that it is in general difficult to obtain an explicit writing of the Poisson cohomology groups of a given Poisson algebra. One of the reasons seems to be that Poisson cohomology is not a functor. A Poisson morphism between two Poisson algebras does indeed not lead to a morphism between their cochains (that is to say the skew-symmetric multiderivations) nor between their Poisson cohomology groups.

The Poisson cohomology has been determined in a few cases of Poisson varieties. For example, for a symplectic manifold, Poisson and de Rham cohomologies are isomorphic, as explained by A. Lichnérowicz in [4], while, for Poisson-Lie groups, one can refer to the works of V. Ginzburg and A. Weinstein, in [5]. In dimension two, the Poisson cohomology was computed in the germified case by P. Monnier and in the algebraic one, by C. Roger and P. Vanhaecke, in [3] and [1].

During my PhD, I determine the Poisson cohomology for some interesting Poisson algebras and I turn these results to good account to study deformations of Poisson structures. I have indeed already obtained an explicit writing of the Poisson cohomology (and homology) of two important classes of Poisson algebras, in two and three dimensions. This work has been the subject of an article published in Comptes Rendus Mathématique and of a manuscript submitted to the Journal of Algebra. Each polynomial φ in $\mathbf{F}[x, y, z]$ defines a Poisson structure on $\mathbf{F}^3$, a surface in $\mathbf{F}^3$ and a Poisson structure on this surface. For all φ that is a weight homogeneous polynomial with an isolated singularity, the surface is a singular one and I have obtained the Poisson cohomology of the two algebras associated to φ. The singularity of the polynomial φ corresponds to the singularity of the surface considered and to the singular locus of the Poisson structure that $\mathbf{F}^3$ is endowed with. I have shown that this singularity, more precisely the algebra of regular functions on this isolated singularity, plays a central role in the Poisson (co)homology of these Poisson algebras. Now, I am working on Poisson structures on singular surfaces in $\mathbf{F}^3$, that are not symplectic on the smooth part of the surface and on Poisson structures in higher dimensions, obtained with an analogous processus.

The further aim of my work is to study deformations of Poisson structures that will need the results of Poisson cohomology that I have obtained.

I hope the contact I will have with Pierre van Moerbeke will enable me to make my research progress.

References

[1] C. ROGER and P. VANHAECKE, *Poisson cohomology of the affine plane*, J. Algebra, **251** : 448–460 (2002)

[2] I. VAISMAN, *Lectures on the geometry of Poisson manifolds*, Progress in Mathematics 118, Birkhäuser, Basel, (1994)

[3] P, MONNIER, *Poisson cohomology in dimension two*, Israel J. Math., **129** : 189–207, (2002)

[4] A. LICHNEROWICZ, *Les variétés de Poisson et leurs algèbres de Lie associées*, J. Differential Geometry, **12** : 253–300, (1977)

[5] V. L. GINZBURG, A. WEINSTEIN, *Lie-Poisson structure on some Poisson Lie groups*, J. Amer. Math. Soc., **5** : 445–453, (1992)

2 Details of the visit

Host

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Period of the visit: 26th of April - 14th of May 2005

Estimated travel costs: 100 euros

Local Expenses: 380 euros (Hotel) + 270 euros (subsistence)