SCIENTIFIC REPORT

Concerning the visit of Spyridon Kamvissis (Greece) to Gerald Teschl, Vienna, Austria, April 18 to July 15, 2005

1. PURPOSE OF THE VISIT

The purpose of the visit has been collaboration concerning (mostly) the asymptotic analysis of the long time behavior of a short range perturbation of the periodic Toda lattice.

2. DESCRIPTION OF THE WORK CARRIED OUT DURING THE VISIT

We have been able to:

(i) Construct a meromorphic Riemann-Hilbert problem for the inverse scattering theory for the short range perturbation of the periodic Toda lattice. Unexpectedly perhaps, we have found that the Riemann-Hilbert problem HAS to be posed on a hyperelliptic Riemann surface of finite genus. Using some symmetry properties we have been able to reduce the problem to a HOLOMORPHIC NORMALIZED (at one of the infinities) Riemann-Hilbert problem.

(ii) Extend the Beals-Coifman-Zhou theory (which relates a holomorphic normalized Riemann-Hilbert problem with any given factorization of the jump matrix to a singular integral equation) to the case of a finite genus Riemann surface. In particular we are replacing the standard Cauchy integral operator by a different integral operator appropriate to the Riemann surface context.

(iii) Locate the stationary phase points expected to be responsible for the dominating contribution to our asymptotic problem.

(iv) Apply the "nonlinear stationary phase" method to extract the long time asymptotics. In particular, we have extended a theorem of Zhou proving existence of a unique solution of a Riemann-Hilbert problem on a finite genus Riemann surface under a certain (Schwarz) symmetry condition on the contour and a definiteness condition on the jump matrix.

3. DESCRIPTION OF THE MAIN RESULTS OBTAINED

We proved that in the absence of eigenvalues of the associated Jacobi operator the perturbed lattice asymptotically approaches the unperturbed periodic lattice. In the presence of eigenvalues one ends up with travelling solitons in a periodic background.

We have proved the equivalence of a holomorphic normalized Riemann-Hilbert problem with a given factorization of the jump matrix on a finite genus Rieman surface to a singular integral equation.

We have proved an existence result for holomorphic normalized Riemann-Hilbert problems on a finite genus Rieman surface, similar to a result of Xin Zhou (1989).

4. PROJECTED PUBLICATIONS/ARTICLES RESULTING FROM OUR GRANT

S.Kamvissis, G.Teschl, Stability of the Periodic Toda Lattice under a Short Range Perturbation.

A first draft already exists.

5. OTHER COMMENTS

Naturally, we expect the collaboration to continue. Future plans include the study of the semiclassical problem and the problem of longer range perturbations. Also, following some recent discussions with Peter Yuditskii (Linz) we believe we may be able to extend the above results to perturbations of almost periodic Jacobi

matrices under the condition that the essential spectrum of the Jacobi matrices is "homogeneous".

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