DESCRIPTION OF PROPOSED PROJECT WORK

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This is a short description of a proposed project work in collaboration with Professor Gerald Teschl of the University of Vienna.

In a recent preprint [1], Egorova, Michor and Teschl have provided a complete theory of direct and inverse scattering for Jacobi operators that are short range perturbations of quasi periodic operators, using and extending the classical results for the constant background case (see, e.g. [5]). Our first goal is to express the inverse scattering problem of [1] as a Riemann-Hilbert factorization problem and use the nonlinear steepest descent method for Riemann-Hilbert factorization problems to extract

(i) long time asymptotics for the (associated) Toda lattice under initial data which are a short range perturbations of quasi periodic coefficients,

(ii) "semiclassical" asymptotics; in other words study the convergence of the dispersive scheme described by the discrete (standard) Toda equations to the solutions of the continuous Toda (partial differential) equations.

The nonlinear steepest descent method method was initiated by Its and made rigorous and systematic by Deift and Zhou. More specifically we will use results and techniques developed by the applicant and collaborators in [2], [3] and possibly [4]. The applicant has recently participated in the formulation of the most general extension of the method, where in fact the term "nonlinear steepest descent method" is given an exact meaning. In analogy with the classical "linear" problem of the asymptotic evaluation of exponential integrals, where a contour deformation reduces the integral to be evaluated into an integral that is explicitly found, [4]

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shows how to deform the original Riemann-Hilbert factorization contour into an appropriate optimal contour which is systematically characterized and computed, on which the resulting Riemann-Hilbert factorization problem is explicitly solved.

Our next goal is to consider infinitely many solitons in a constant background. Our long term goal is to weaken the decay assumption, extend these results to slowly decaying tridiagonal Jacobi operators, and use the associated Riemann-Hilbert factorization problem to extract long time and semiclassical asymptotics for the Toda lattice with slowly decaying data. This is an important extension of the existing integrable theory, which involves new interesting solutions and phenomena (see for example the work of V. Matveev). We believe that the problem above will be a first useful step.

A second immediate and related goal is the study of the Riemann-Hilbert problem formulated by Volberg and Yuditskii [6] for a different but very broad class of tridiagonal Jacobi operators which can give rise to a spectrum with a Cantor set as support! The understanding of the Riemann-Hilbert problem will be of great importance for the study of semiclassical asymptotics. We expect to observe some irregular behavior in the semiclassical limit that is not covered by the existing finite-gap theory. In this direction, we also expect some collaboration with P. Yuditskii who is visiting Linz in 2005.

REFERENCES

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